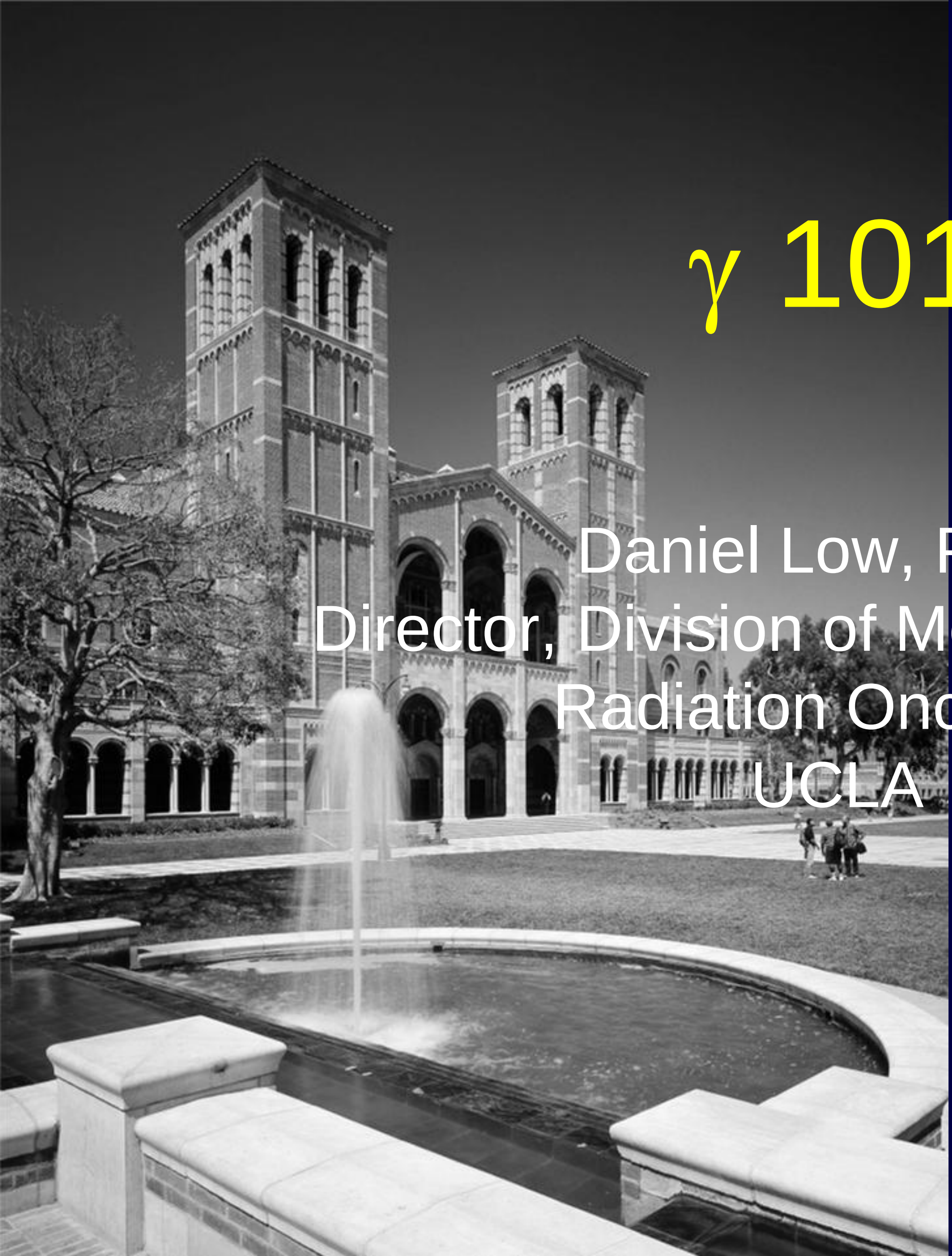
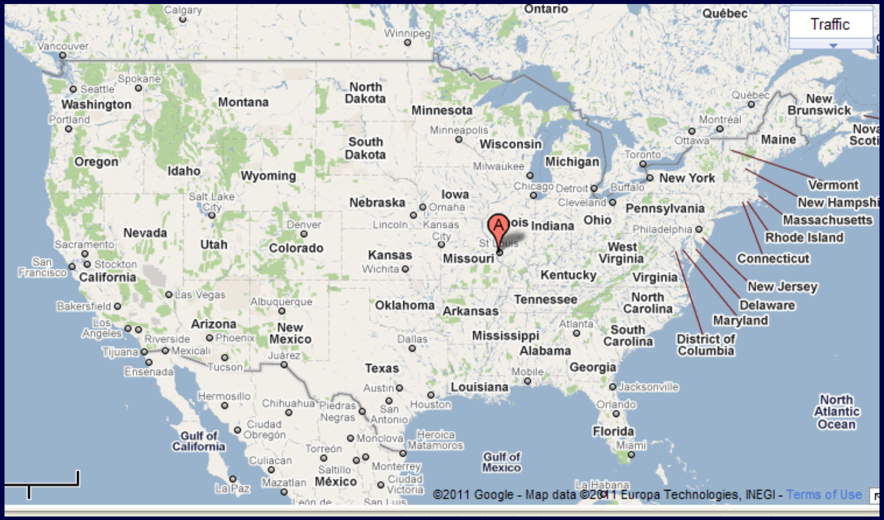


γ 101

Daniel Low, Ph.D.
Director, Division of Medical Physics
Radiation Oncology
UCLA

Decisions - Before



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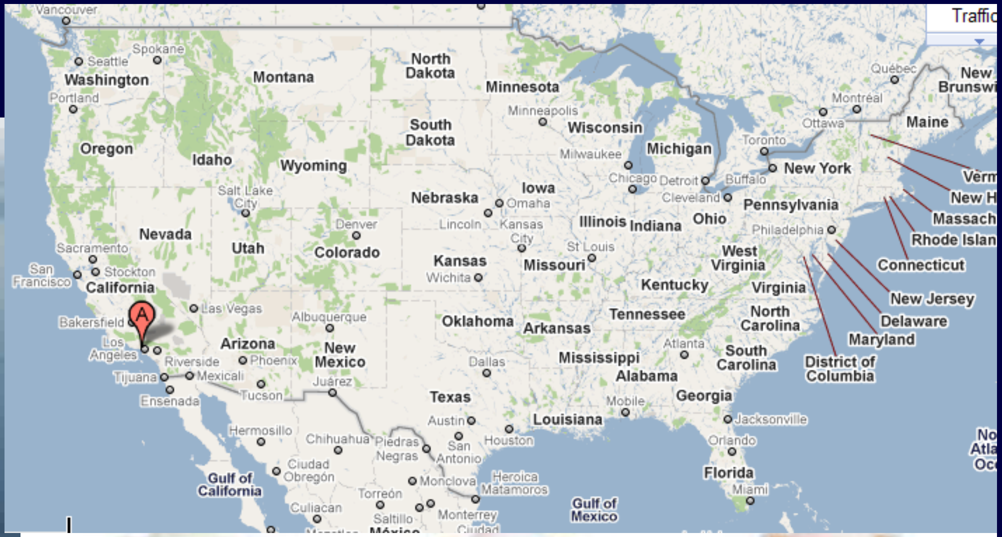
Decisions - Now



What is γ?

- γ is the rescaled minimum Euclidean distance between an evaluated distribution and each point in a reference distribution
- Each spatial and dose axis is normalized by a criterion
- Renormalized “distance” defaults to distance to agreement and dose difference in shallow and steep dose gradient regions, respectively.

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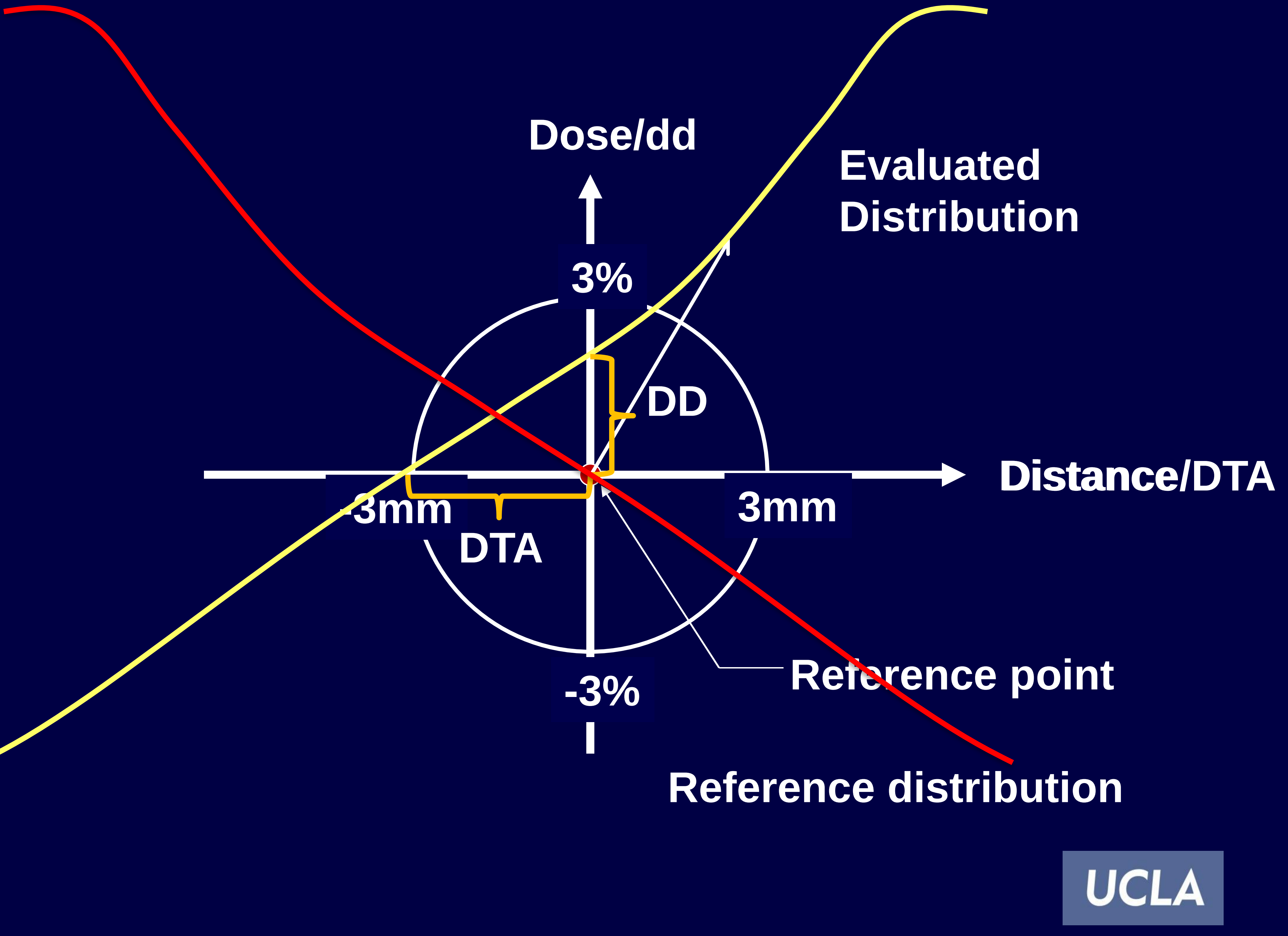
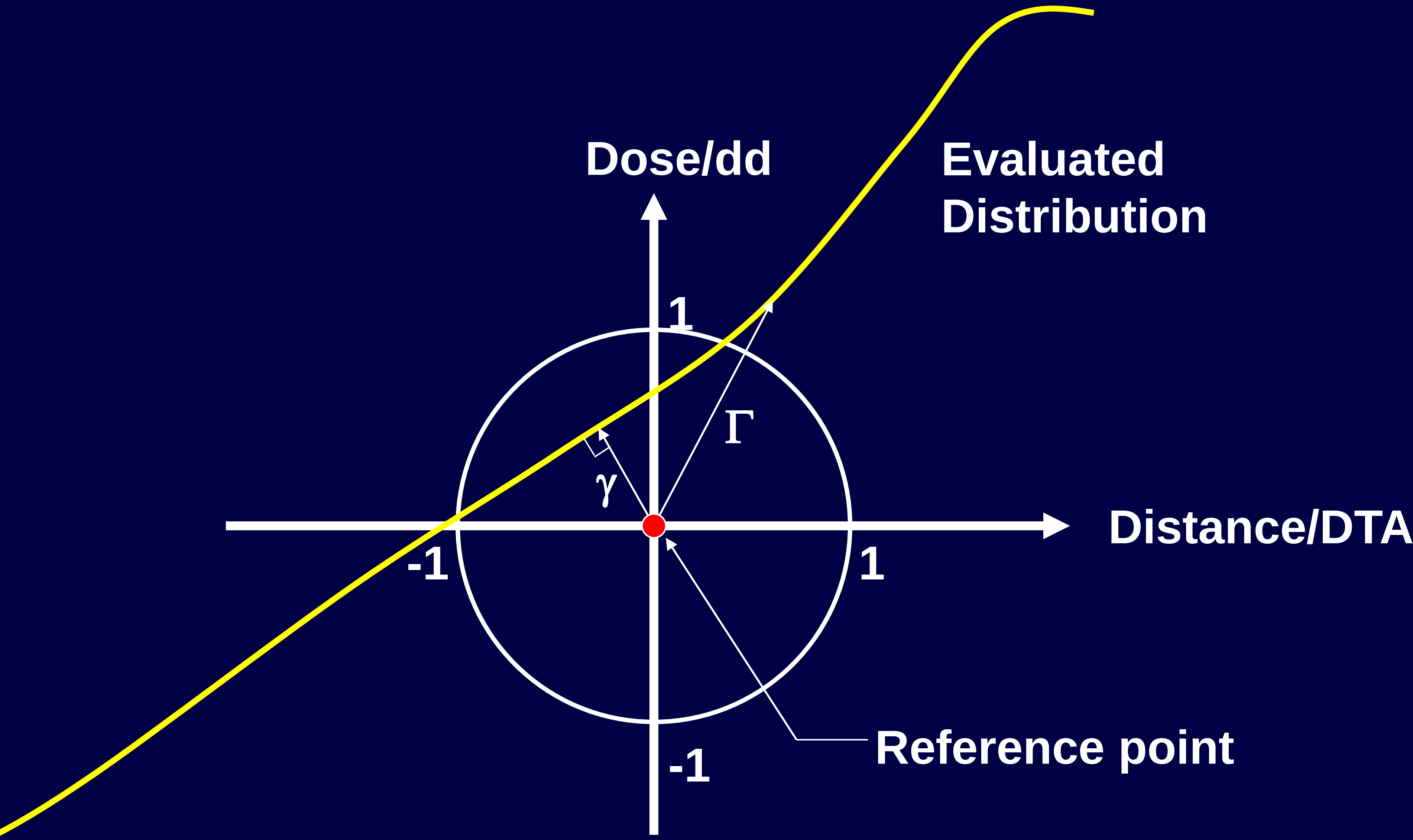
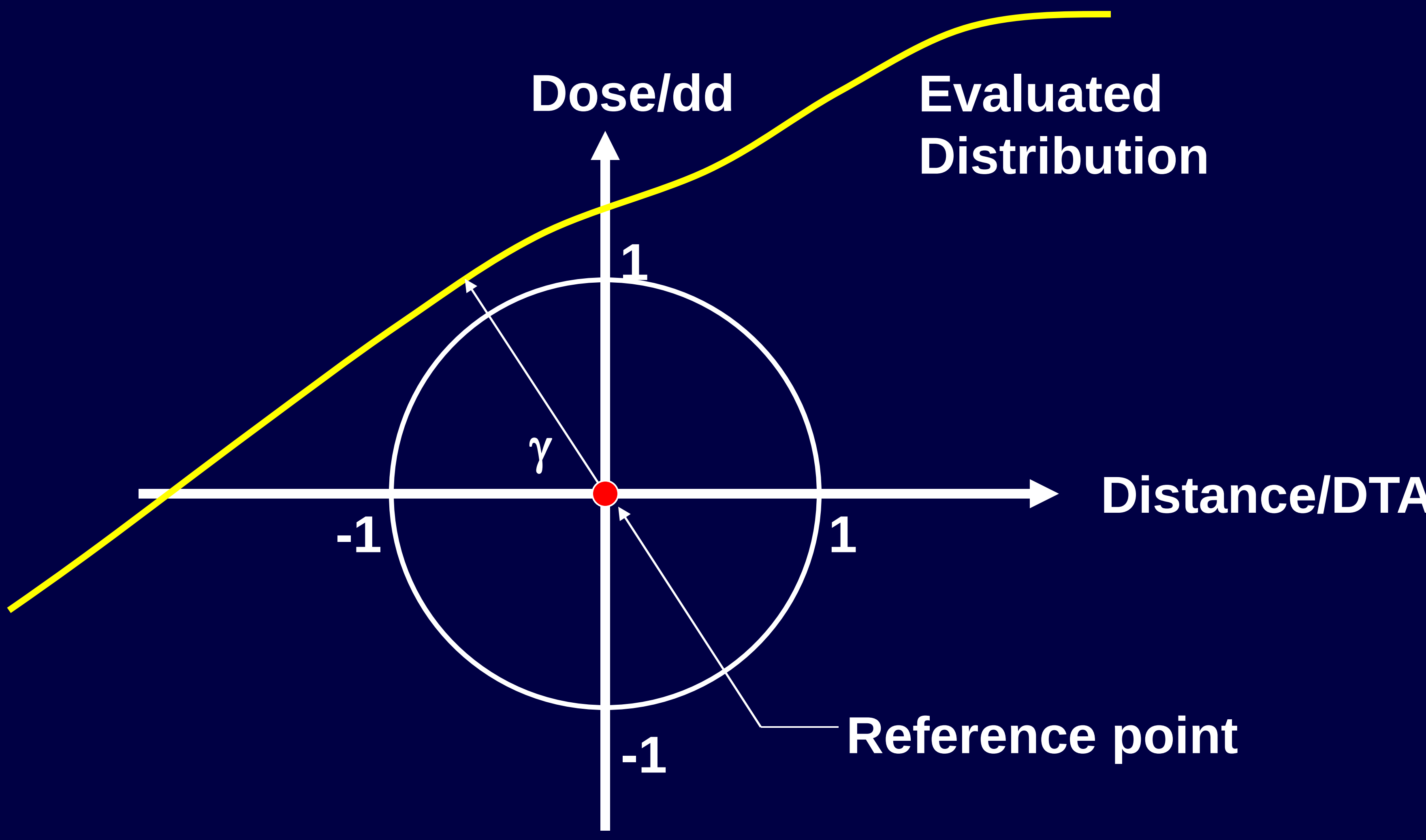
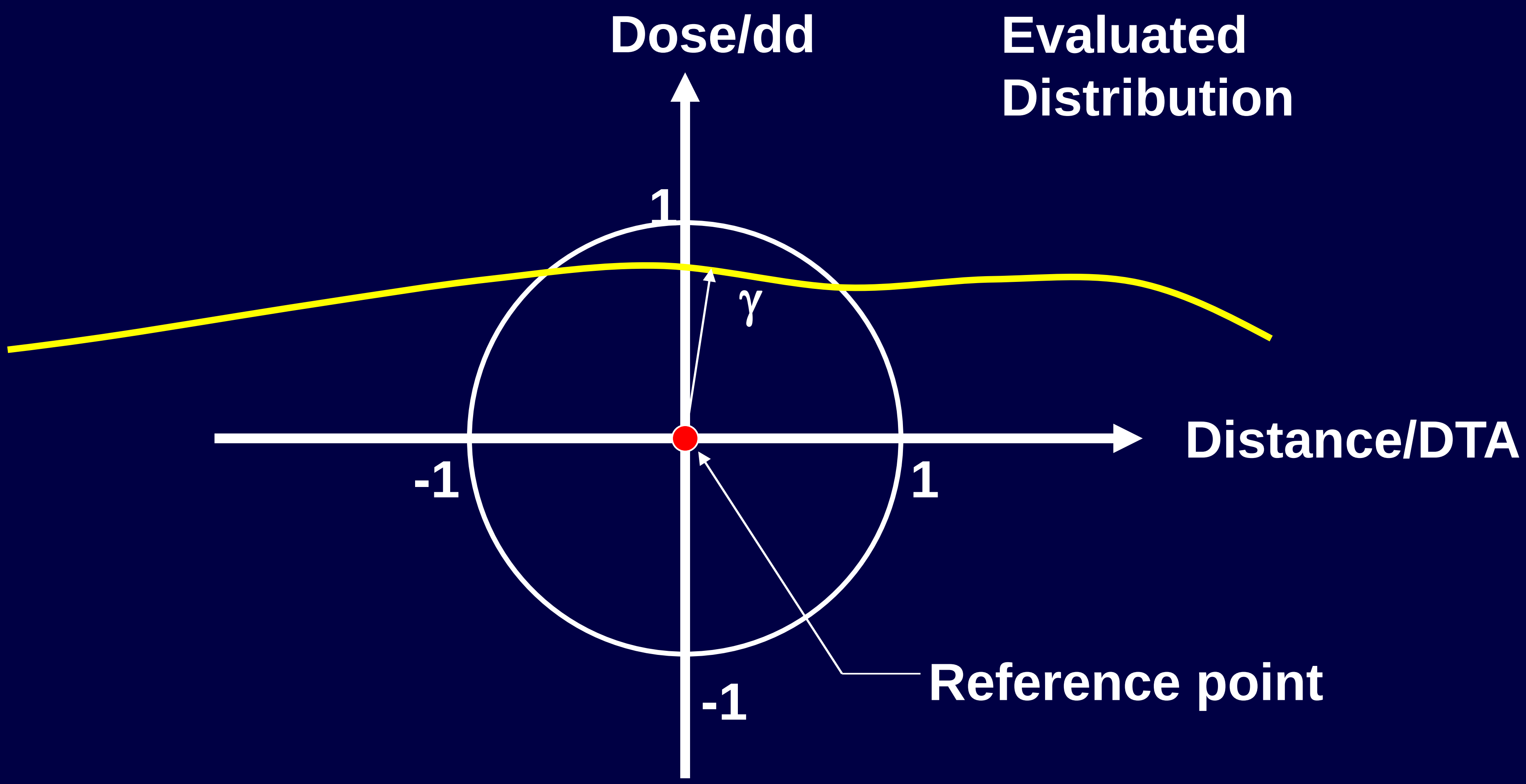
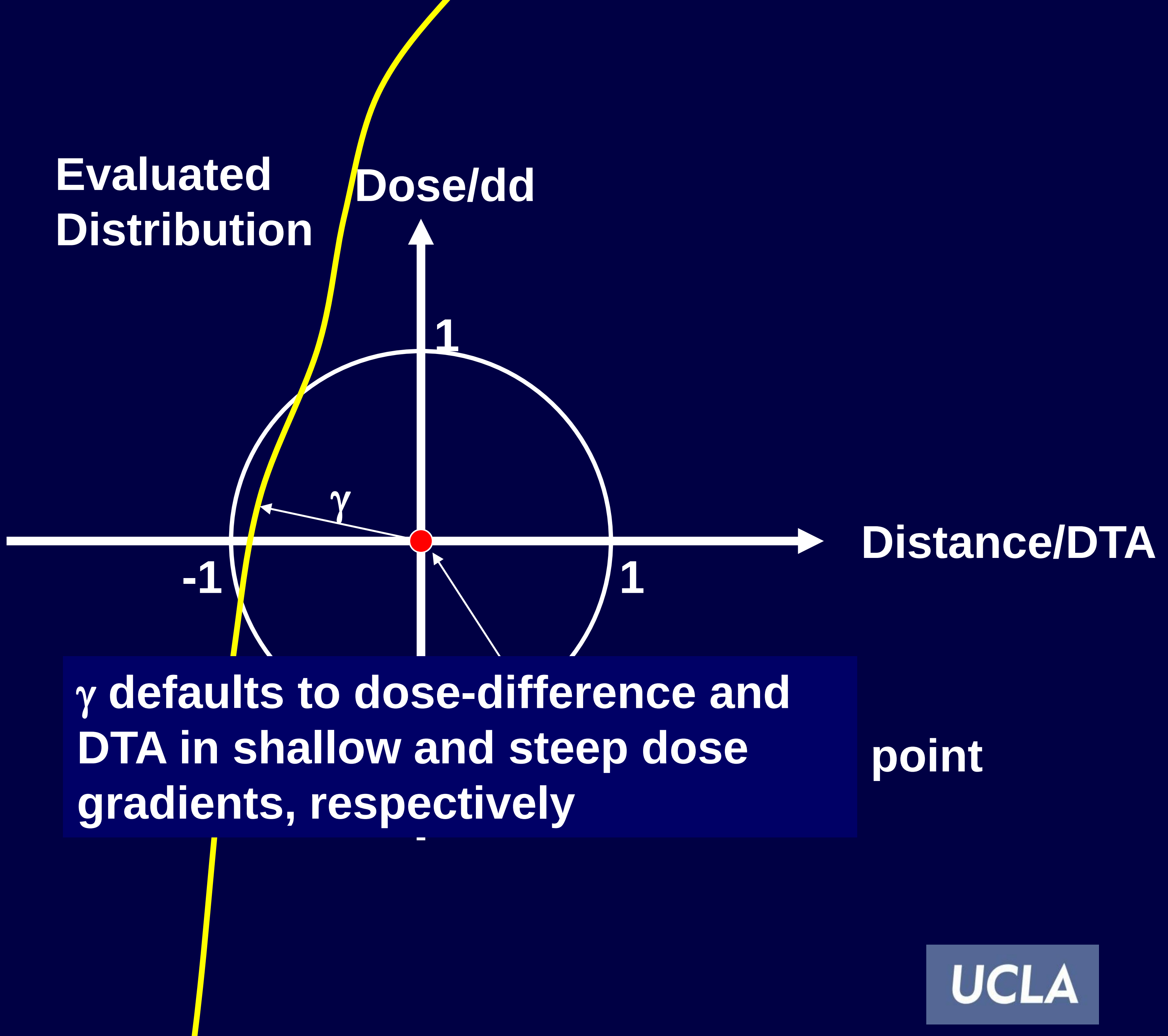

Dose/dd
Evaluated
Distribution
3%
DD
Distance/DTA
-3mm
3mm
DTA
-3%
Reference point
Reference distribution
UCLA


Dose/dd
Evaluated Distribution
1
Γ
γ
Distance/DTA
-1
1
Reference point
-1

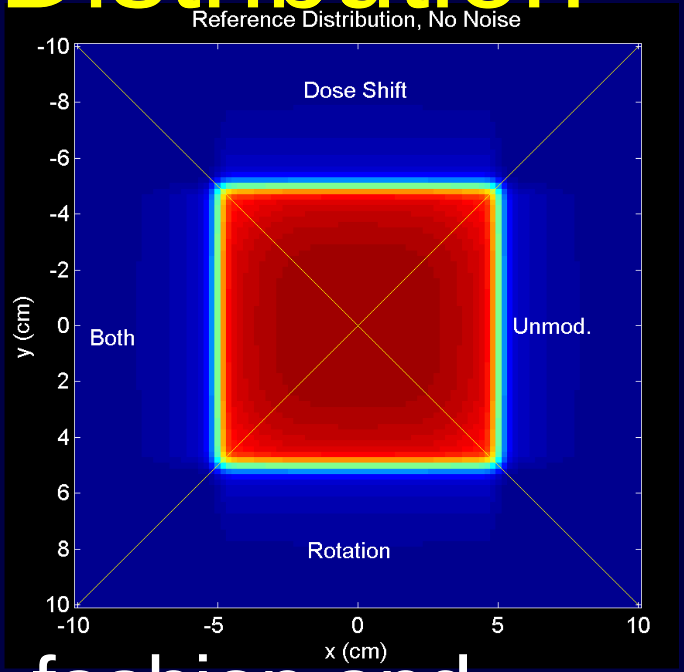
UCLA


Dose/dd
Evaluated
Distribution
1
γ
Distance/DTA
-1
1
Reference point
-1
UCLA


Dose/dd
Evaluated
Distribution
1
γ
Distance/DTA
-1
1
Reference point
-1
UCLA


Evaluated
Distribution
Dose/dd
1
γ
-1
1
Distance/DTA
γ defaults to dose-difference and DTA in shallow and steep dose gradients, respectively
point
UCLA

Example Dose Distribution



- Two 10 x 10 fields
- 6 MV
- Coronal
- 3%, 3mm criteria
- Skew one in a smooth fashion and compare doses

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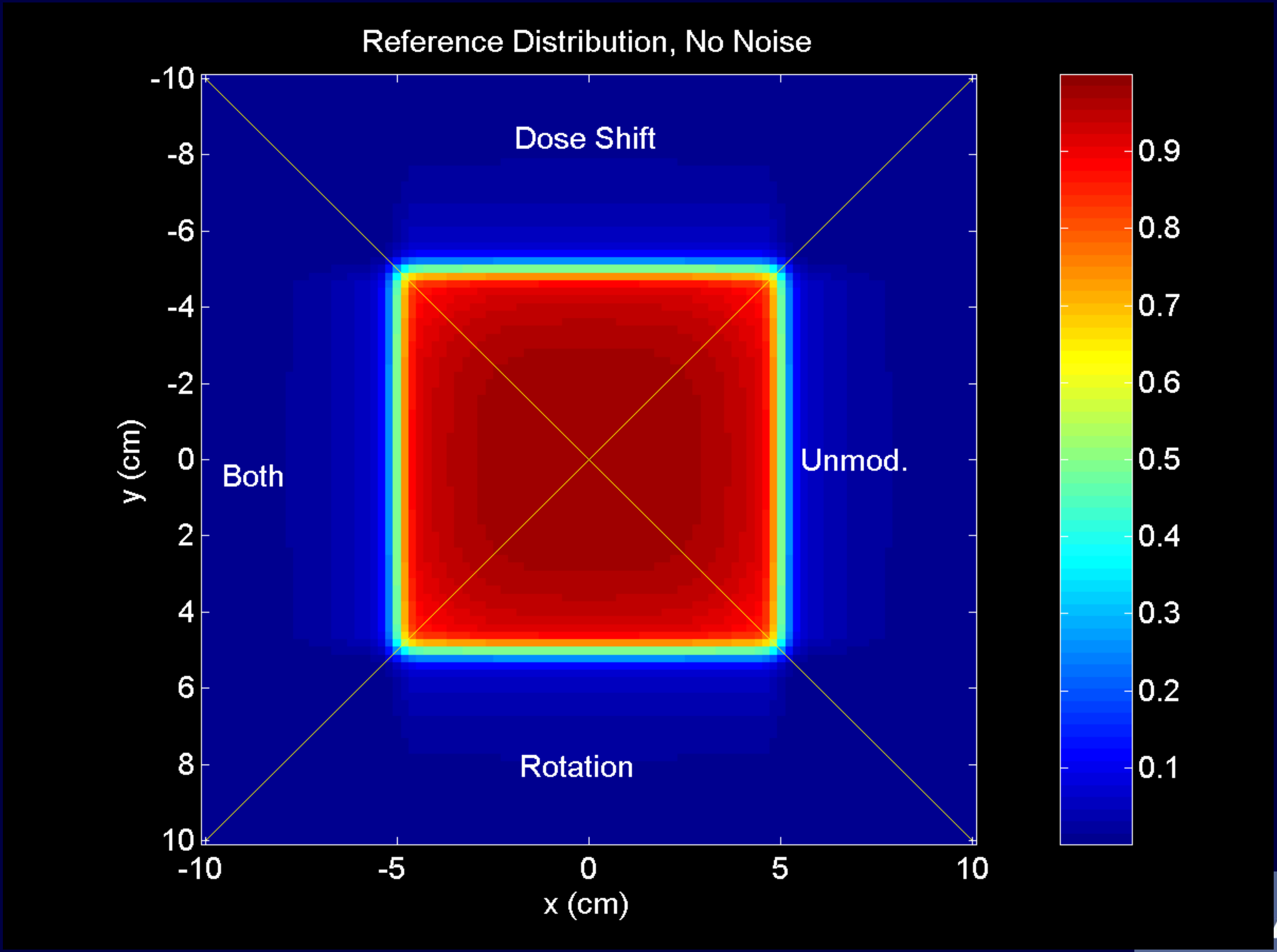
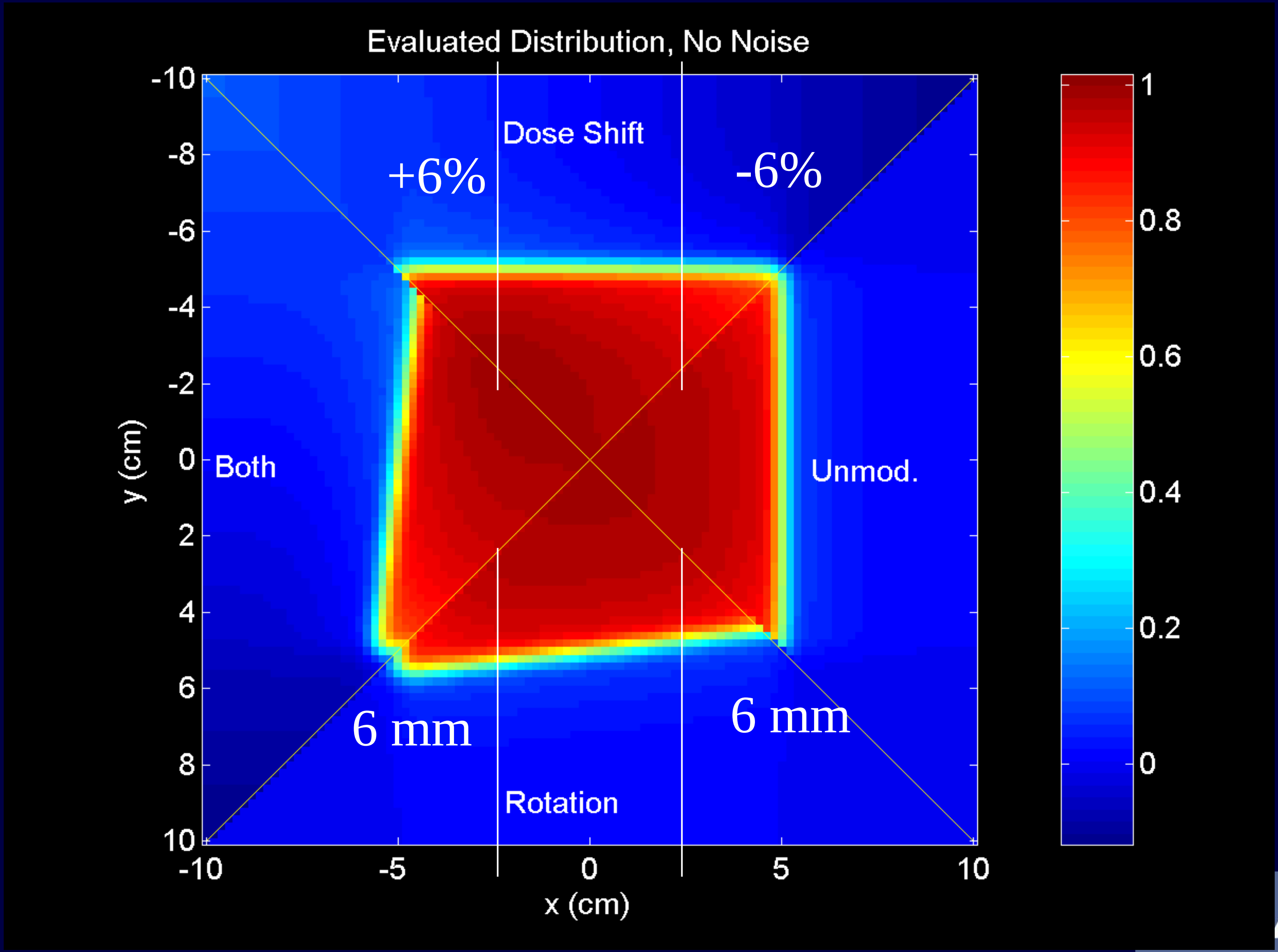
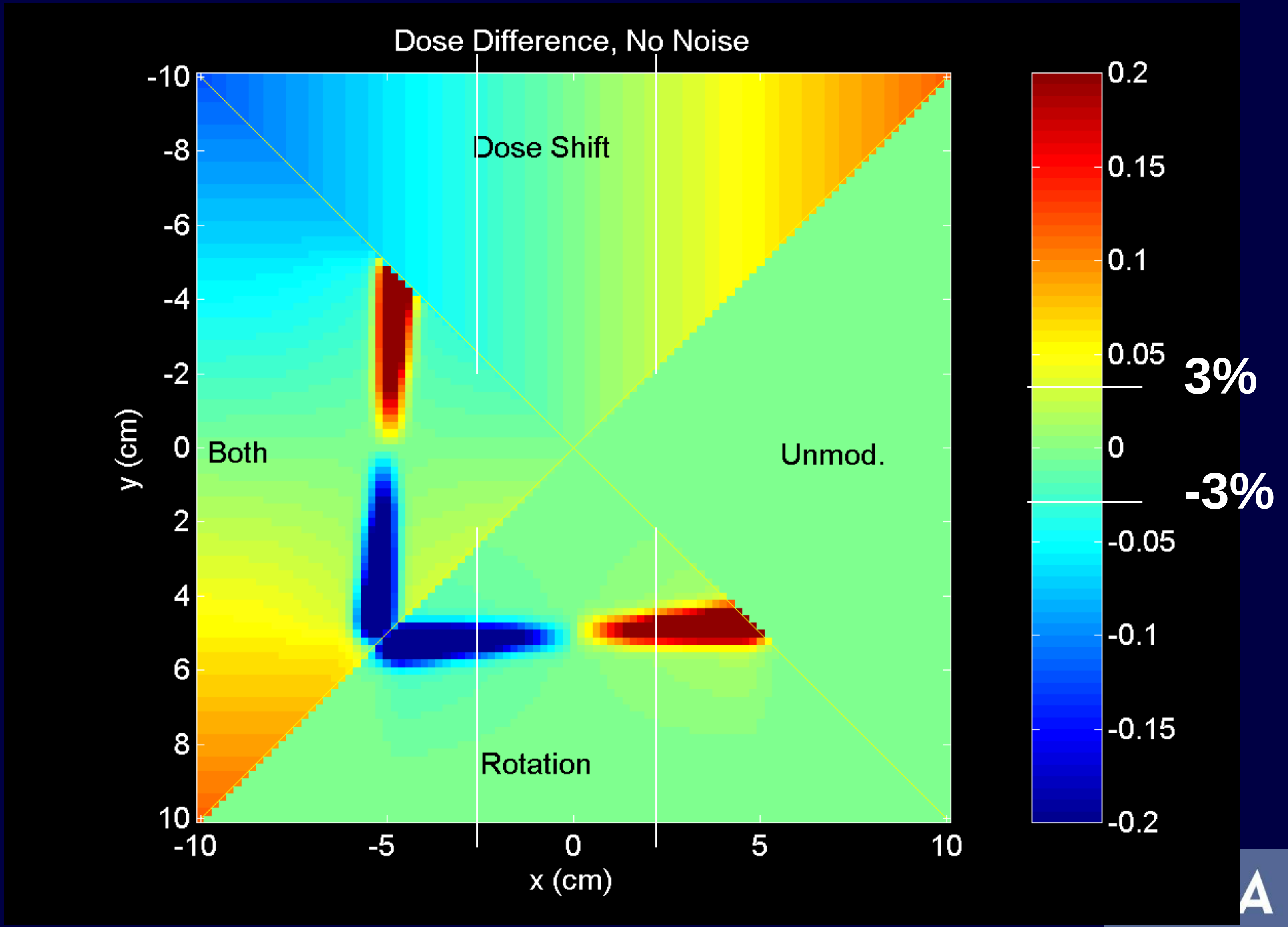
Reference Distribution (10x10 cm^2)



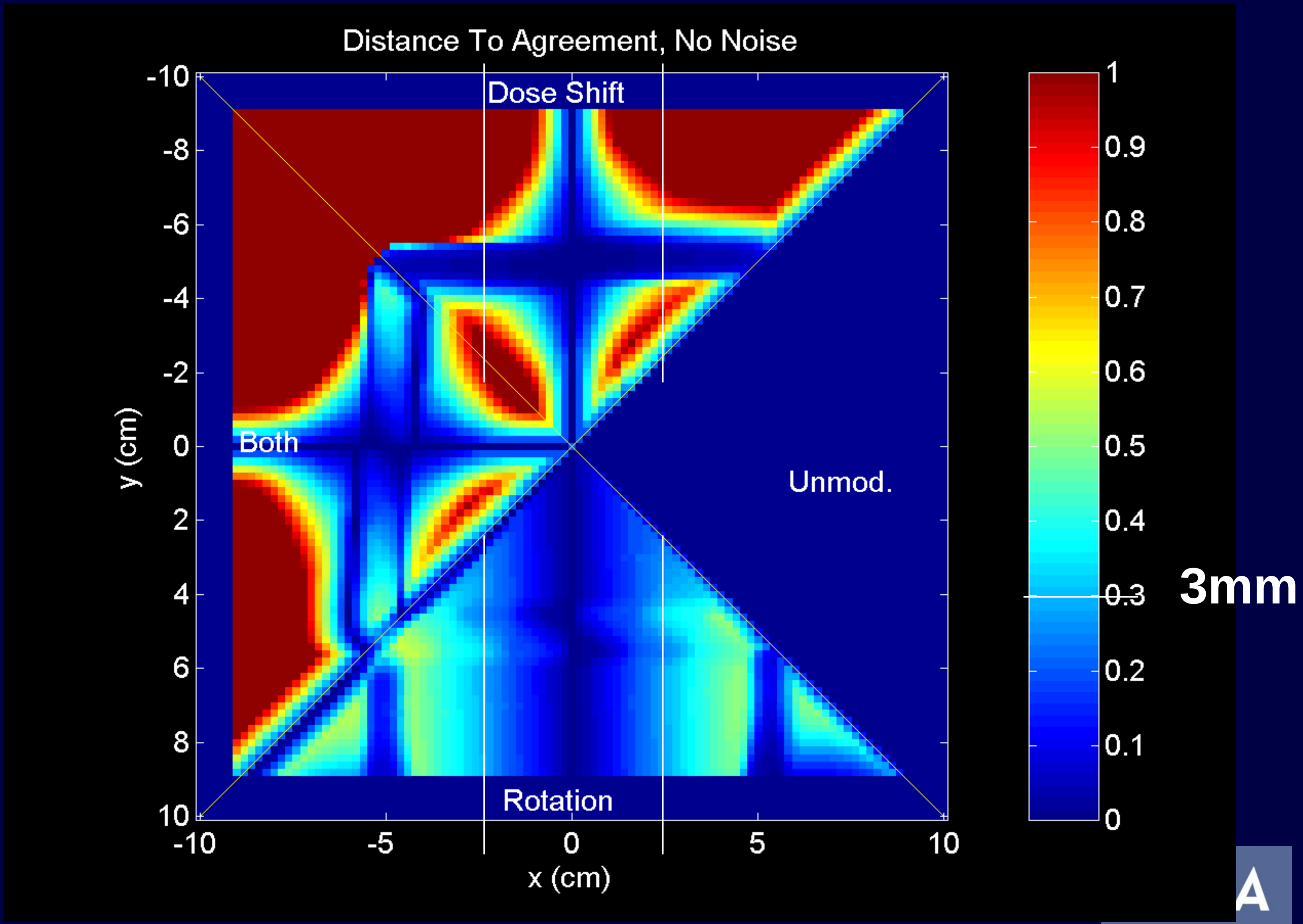
Evaluated Distribution



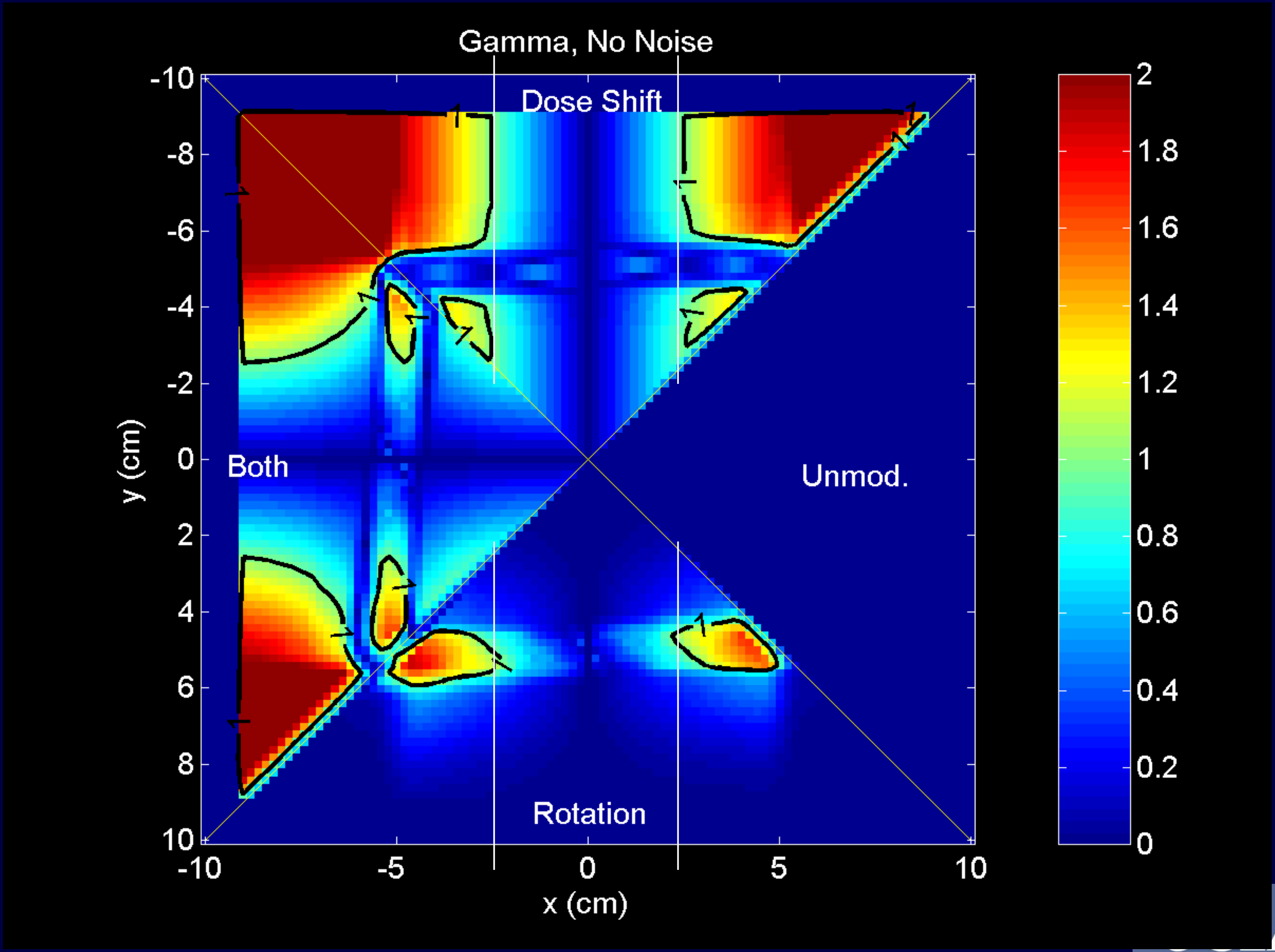
Dose Difference



Distance-to-Agreement



Gamma



Gamma, No Noise
Dose Shift
Both
Unmod.
Rotation
y (cm)
x (cm)
-10
-8
-6
-4
-2
0
2
4
6
8
10
-10
-5
0
5
10
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0

Spatial Resolution

- γ is calculated independently for each reference point
- Reference distribution can be a single point
- Evaluated distribution 1D-3D
- Resolution challenge

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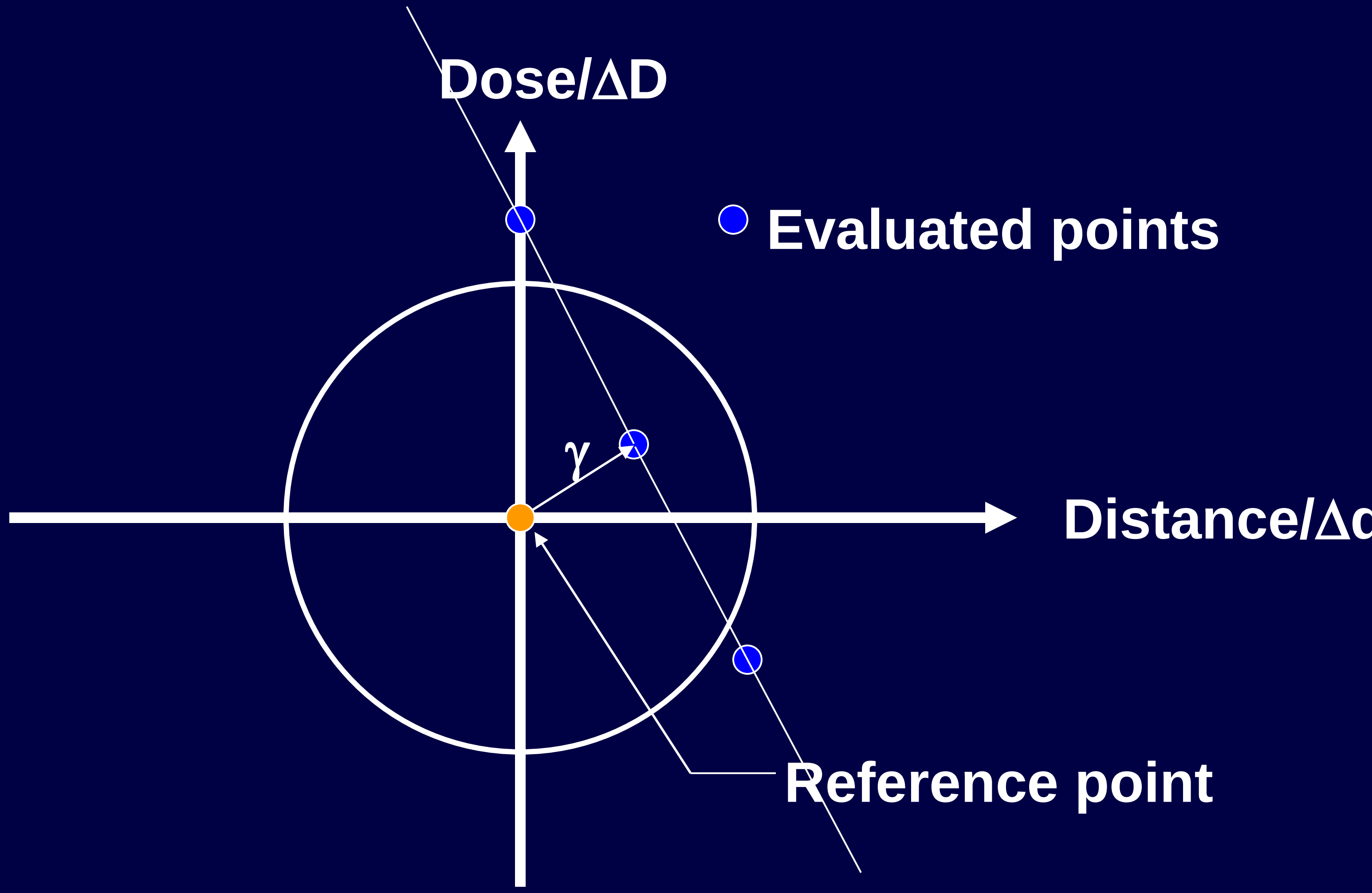
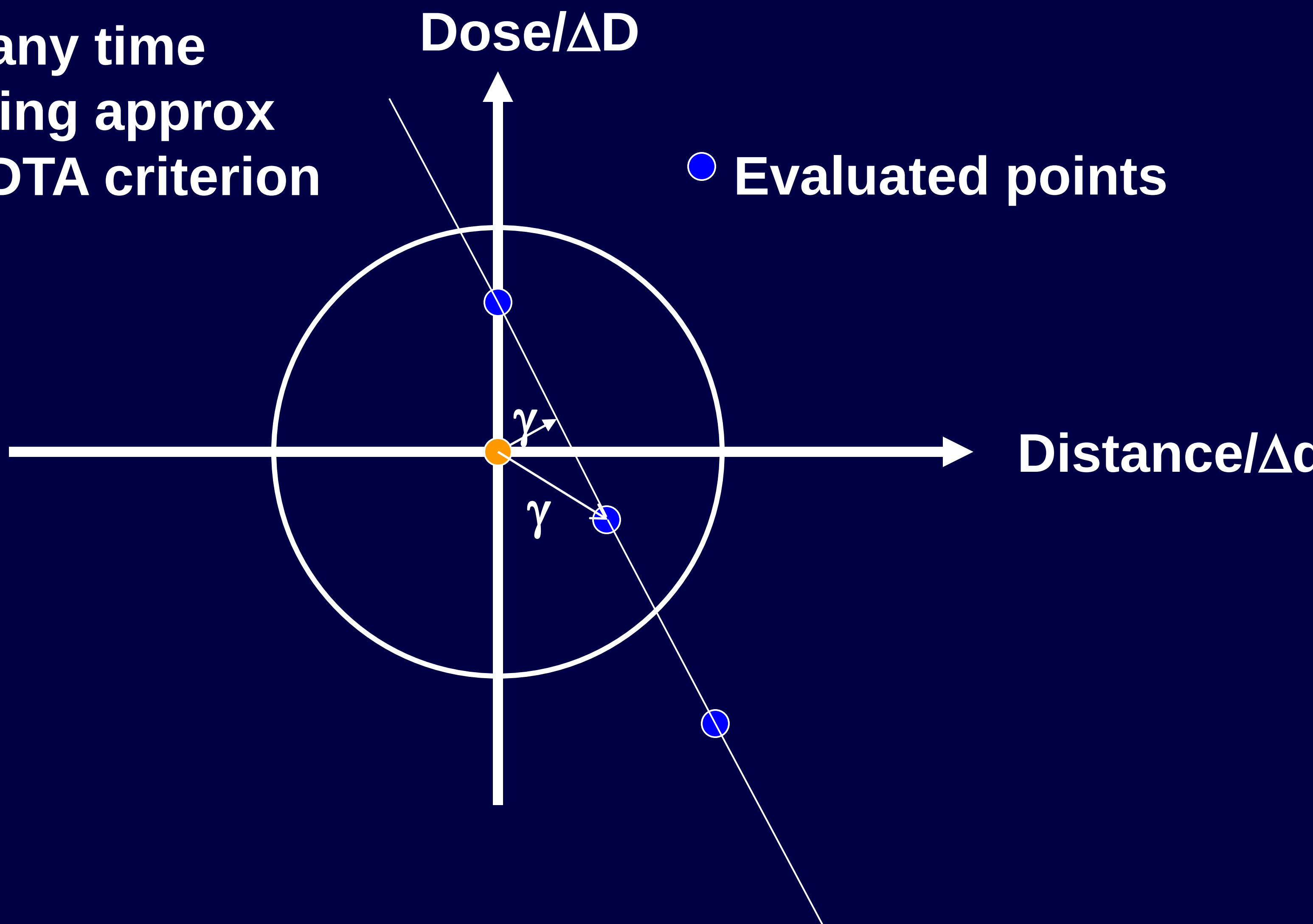
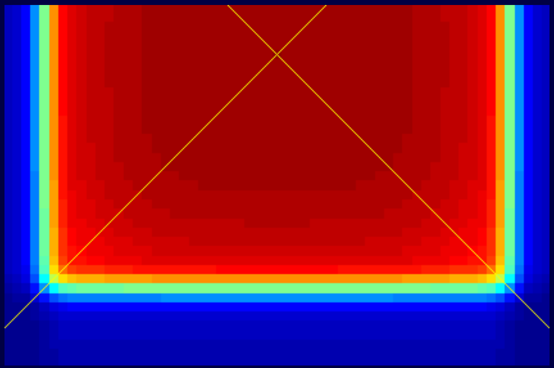
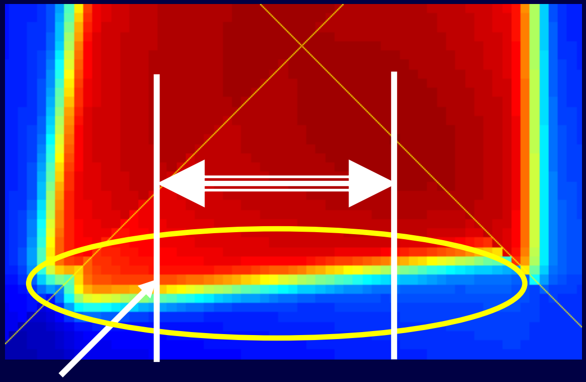

Dose/ΔD
Evaluated points
γ
Distance/Δd
Reference point

Problem any time eval spacing approx same as DTA criterion

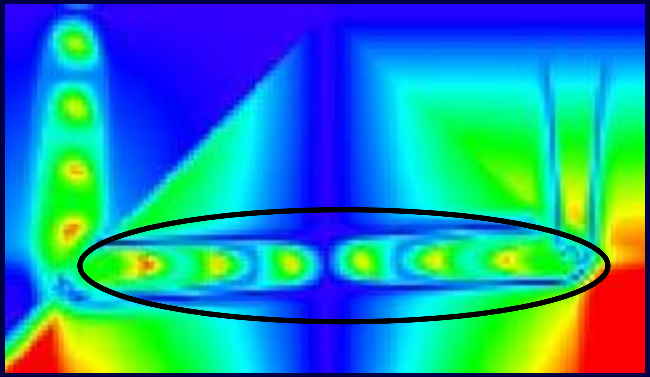


Evaluation Distribution Interpolation



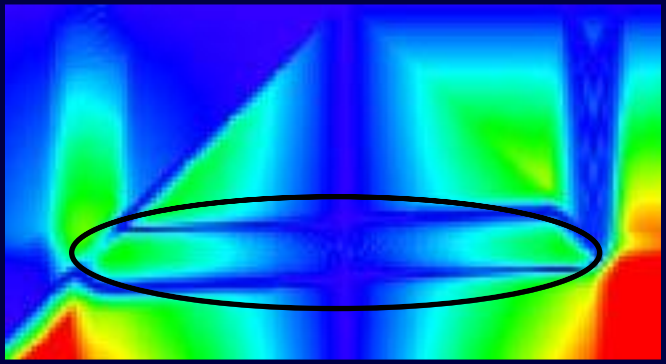


3% & 3mm



Uninterpolated

γ



Interpolated voxels 8x

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Interpolation

- Fixes resolution problem with evaluated distribution
- Cost in computation time
- Think of interpolation as geometric problem
  - Closest distance between line, surface, volume and one point
- Fast computation provided by computer gaming
- Ju et al. Med. Phys. 35, 879-887 (2008).

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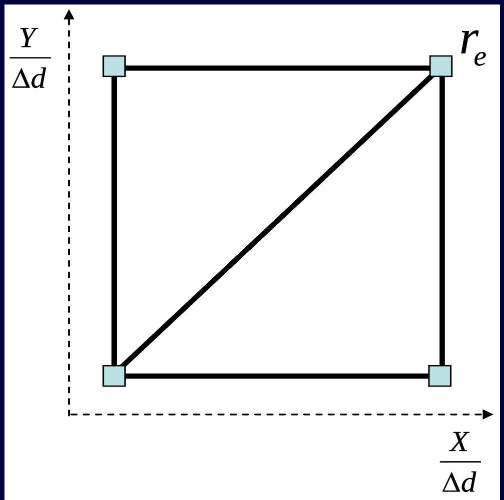
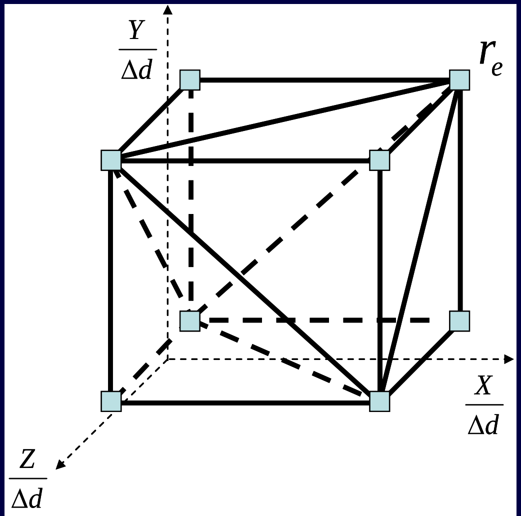
Simplexes

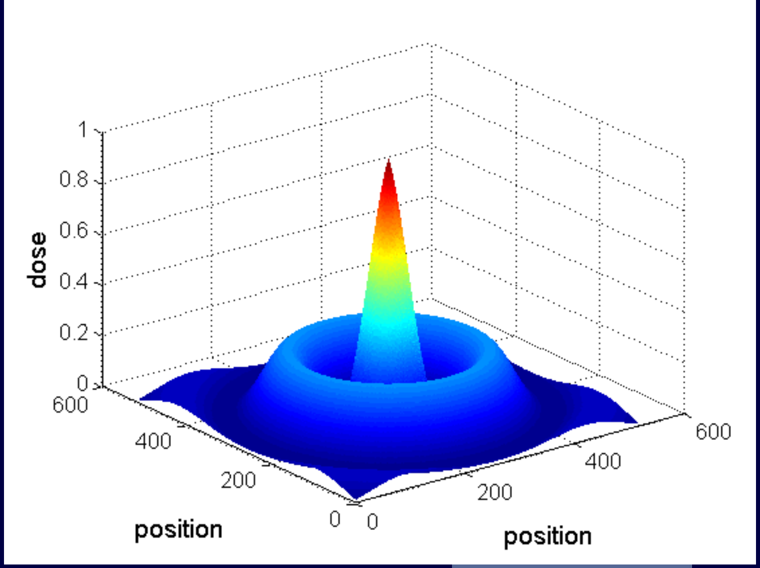
- Break up evaluated dose distribution into simplexes
  - Line segments, triangles, tetradhedra
  - Distance from point to a simplex can be computed easily in closed form
- Making Simplexes
  - Quadrilateral dose surface divided into two triangles
  - Cubic (hexahedral) dose surface divided into five tetrahedra.





Distance Calculation

- Make dose surface into simplicial mesh
  - Collection of Simplexes
    - 3D for film, 4D for dose distribution comparisons (n)
    - k-simplex S (0<= k <= n) is convex hull of k +1 points (vertices of S)
      - Film dosimetry, simplex can be point (0-simplex), line (1-simplex), or triangle (2-simplex)
- We want distance between p and the evaluated distribution



Distance Calculation

- Distance from p to k-simplex S is shortest distance between p and
  - Any point lying on boundary or interior of S
- At any boundary or interior point v of S can be described by

$$v = \sum_{i=1}^{k+1} w_i v_i$$

- Where $\{v_1, \ldots, v_{k+1}\}$ are coordinates of S vertices and $\{w_1, \ldots, w_{k+1}\}$ are non-negative weights from zero to 1 and sum up to 1.

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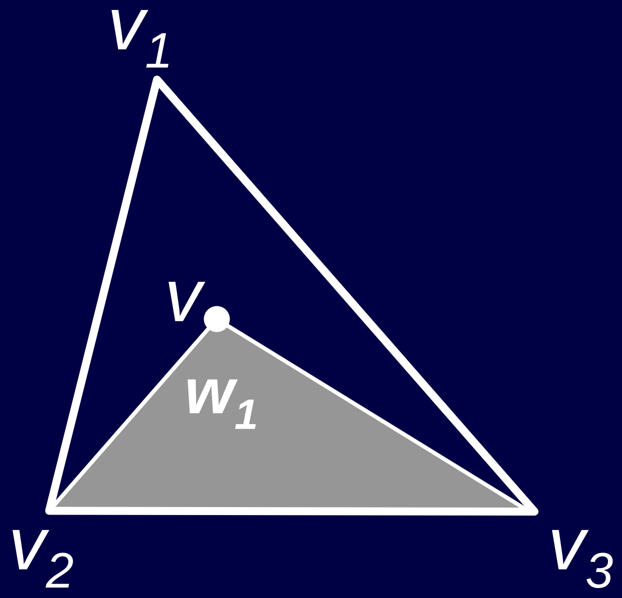
Weight Point Definition

- Weights w are the relative {length, area, volume, hypervolume} of the opposing simplex.
- 2-simplex: the three weights add to 1



2-simplex

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Compute Distance to Simplex S

- Solve following (point on plane relative to reference point)

$$\overline{D}(p,S) = \min_{\{w_1,\ldots,w_{k+1}\}, s.t. \sum_{i=1}^{k+1} w_i = 1} | p - \sum_{i=1}^{k+1} w_i v_i |$$

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Solution

- This equation has a closed form solution that requires ONLY matrix inversion!

$$\{w_1,\ldots,w_k\} = (V^T V)^{-1} V^T P, \quad w_{k+1} = 1 - \sum_{i=1}^{k} w_i$$

- Where

$$P = \begin{Bmatrix} c_1(p) - c_1(v_{k+1}) \\ \vdots \\ c_n(p) - c_n(v_{k+1}) \end{Bmatrix}, \quad V = \begin{Bmatrix} c_1(v_1) - c_1(v_{k+1}) & \cdots & c_1(v_k) - c_1(v_{k+1}) \\ \vdots & \vdots & \vdots \\ c_n(v_1) - c_n(v_{k+1}) & \cdots & c_n(v_k) - c_n(v_{k+1}) \end{Bmatrix}$$

- c_i refers to the *i*th coordinate (e.g. x, y, d)

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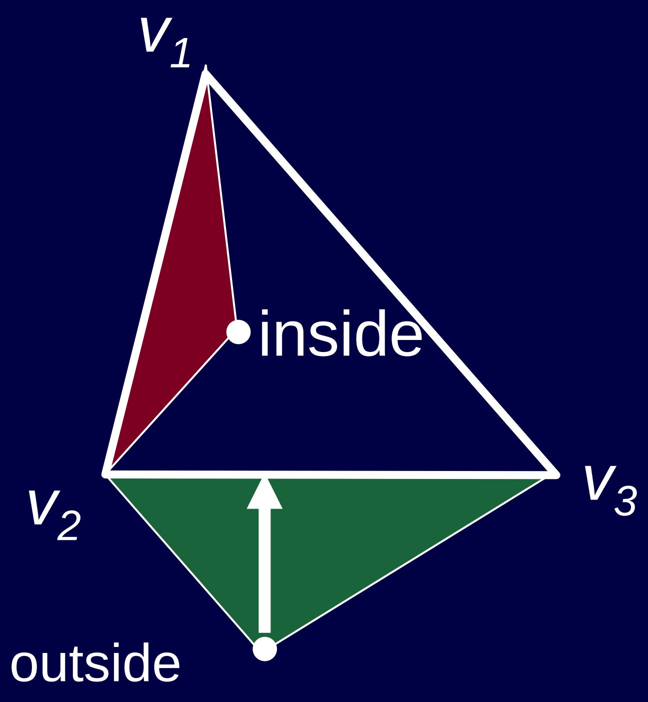
Locate Minimum Distance

- The minimum distance either lies
  - Within the simplex
    - Done!
  - Outside the simplex
    - Must lie on the edge or vertex of the simplex
    - This is just the next lower dimension simplex!
    - Recursive algorithm



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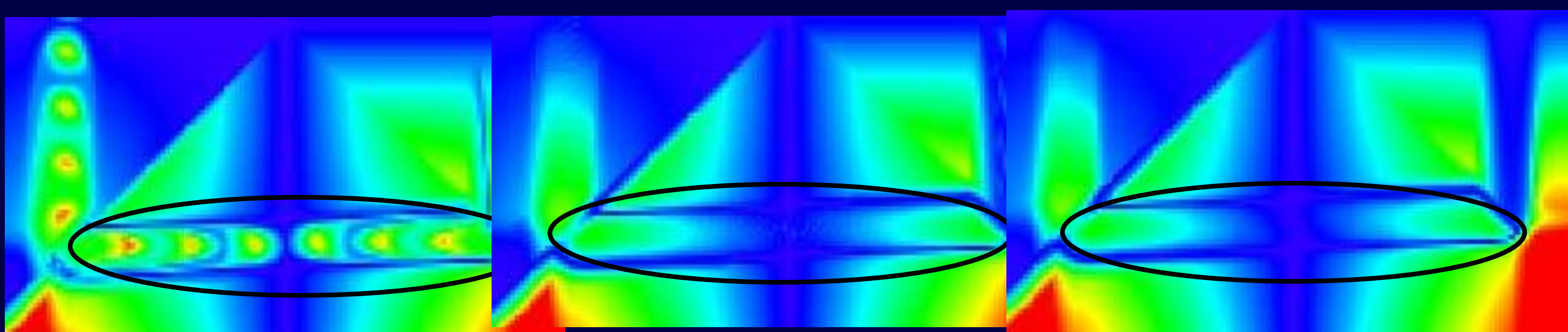
Results



Uninterpolated

Interpolated voxels 8x

Geometric Method

Noise and γ

- Dose distribution noise has profound impact on γ calculations
- The impact depends on whether the noise is in the reference or evaluated distributions

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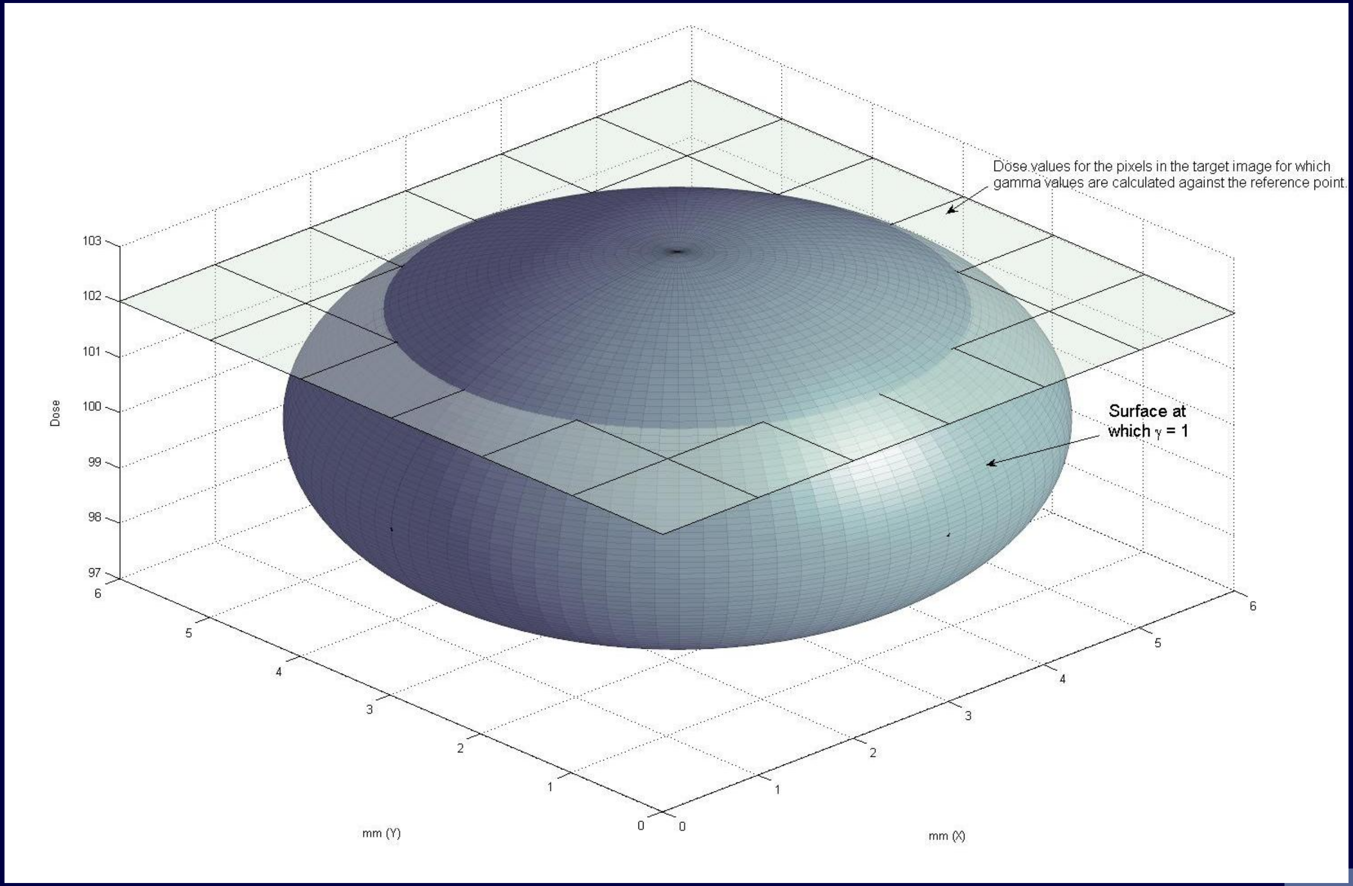
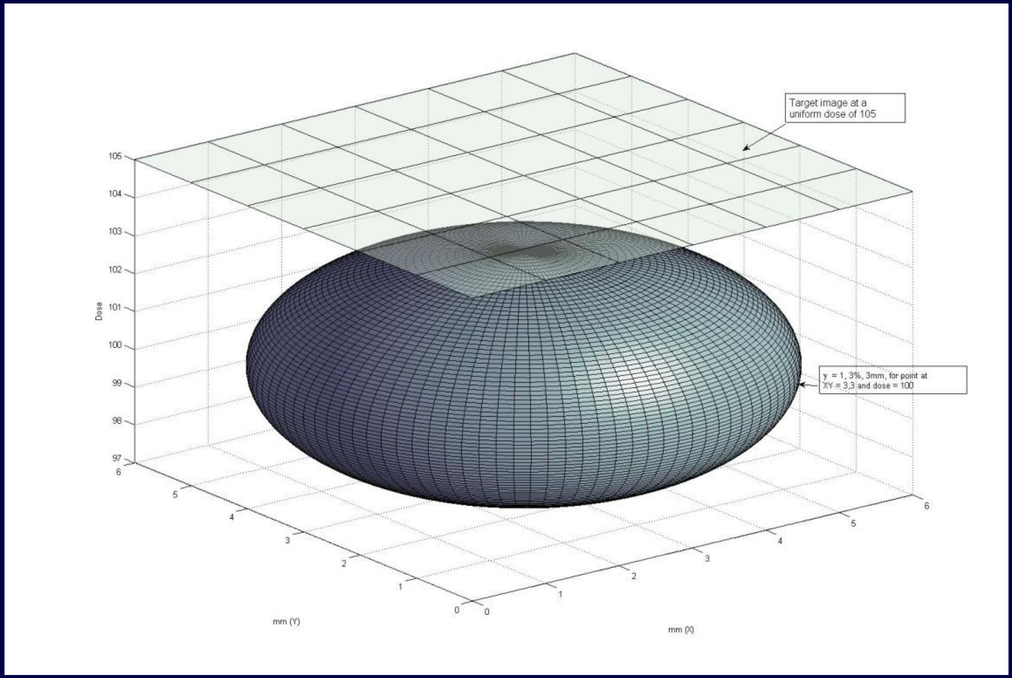
Why Noise Impacts γ



Thanks to Matt Whitaker, RIT

- Ideal case with a constant 5% difference between the point to be evaluated and the target image surface.
- With no noise a 3mm, 3% gamma will evaluate to 1.667 for this situation (fail).



Thanks to Matt Whitaker, RIT

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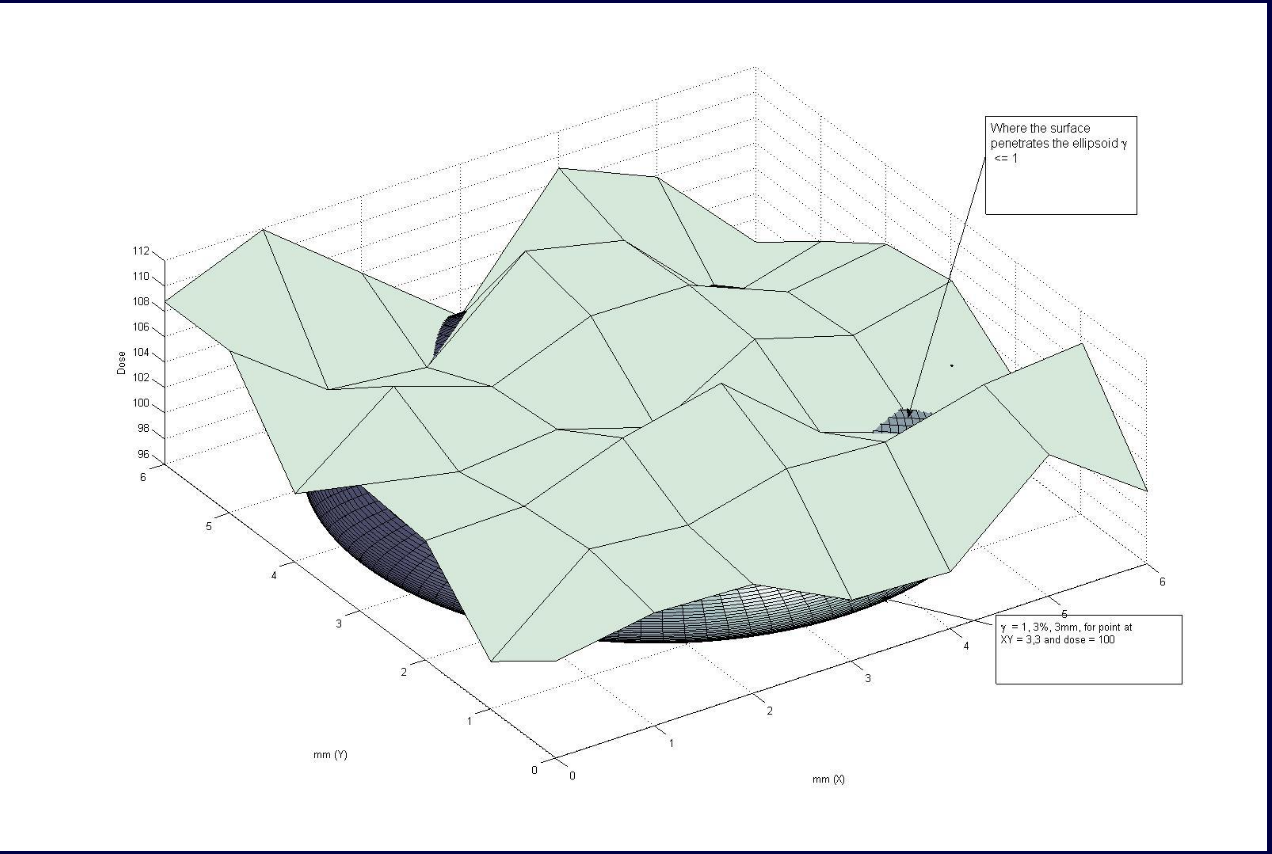
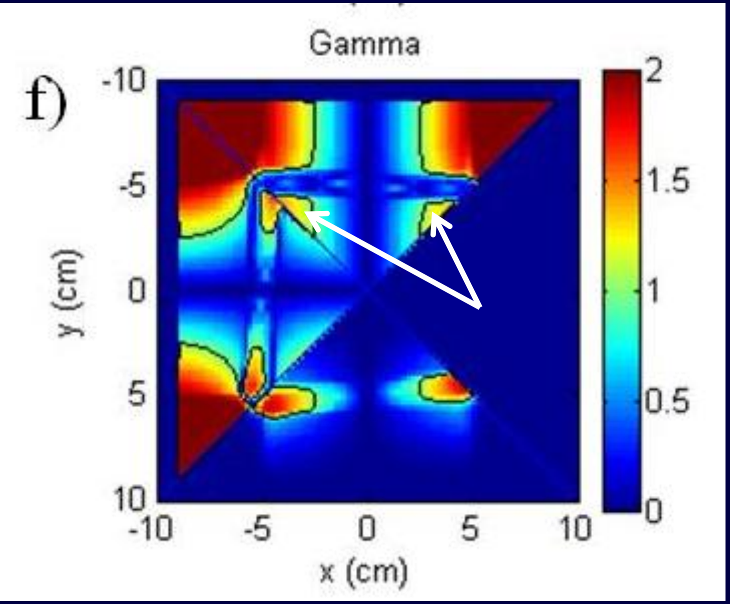
- If we add Gaussian noise with 0 mean and 3.16 standard deviation we see that the ellipsoid is penetrated.
- Anywhere the ellipsoid is penetrated γ <= 1 (pass)



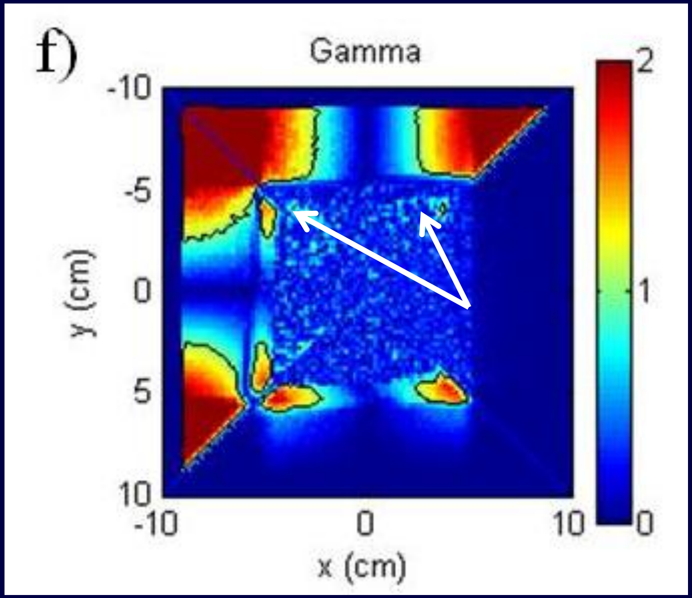
Thanks to Matt Whitaker, RIT

- Impact of noise depends on whether it is in the reference or evaluated distribution!
  - Evaluated: Typically underestimates γ (γ is the **minimum** distance!)
  - Reference: Noise is reflected in γ

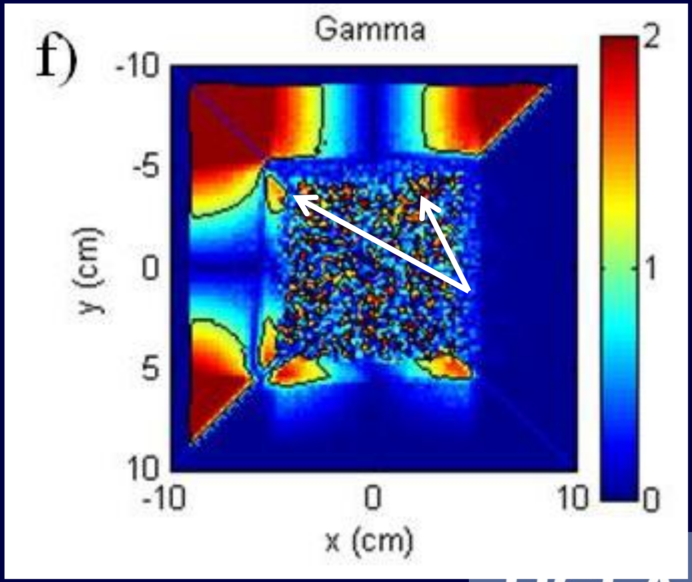
No Noise



3% Evaluated



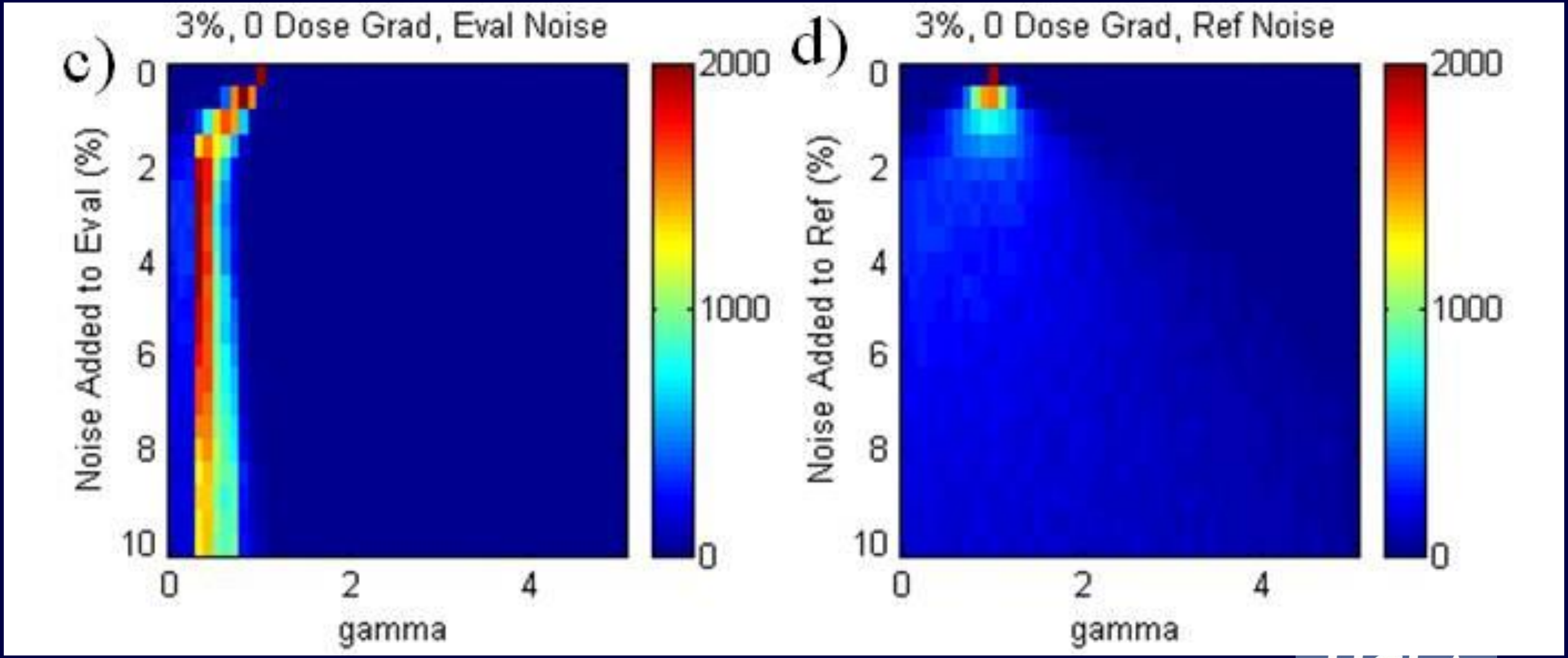
3% Reference



Low and Dempsey Med. Phys. 30, 2455-2464 (2003).

Gamma Histograms

- Two distributions with no dose gradients (flat). Differ by 3% (3% dd criterion) so no-noise γ is 1 everywhere



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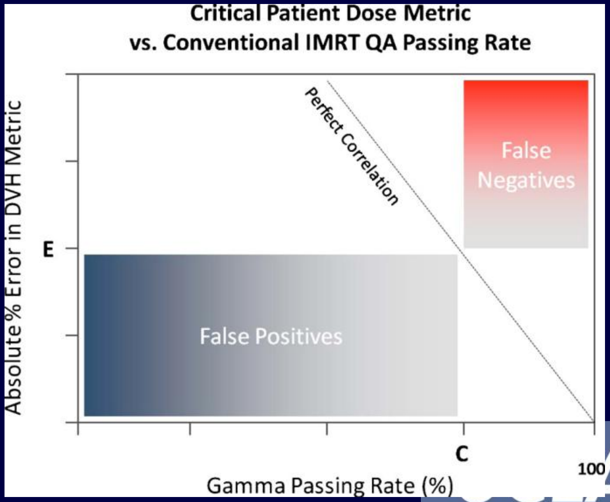
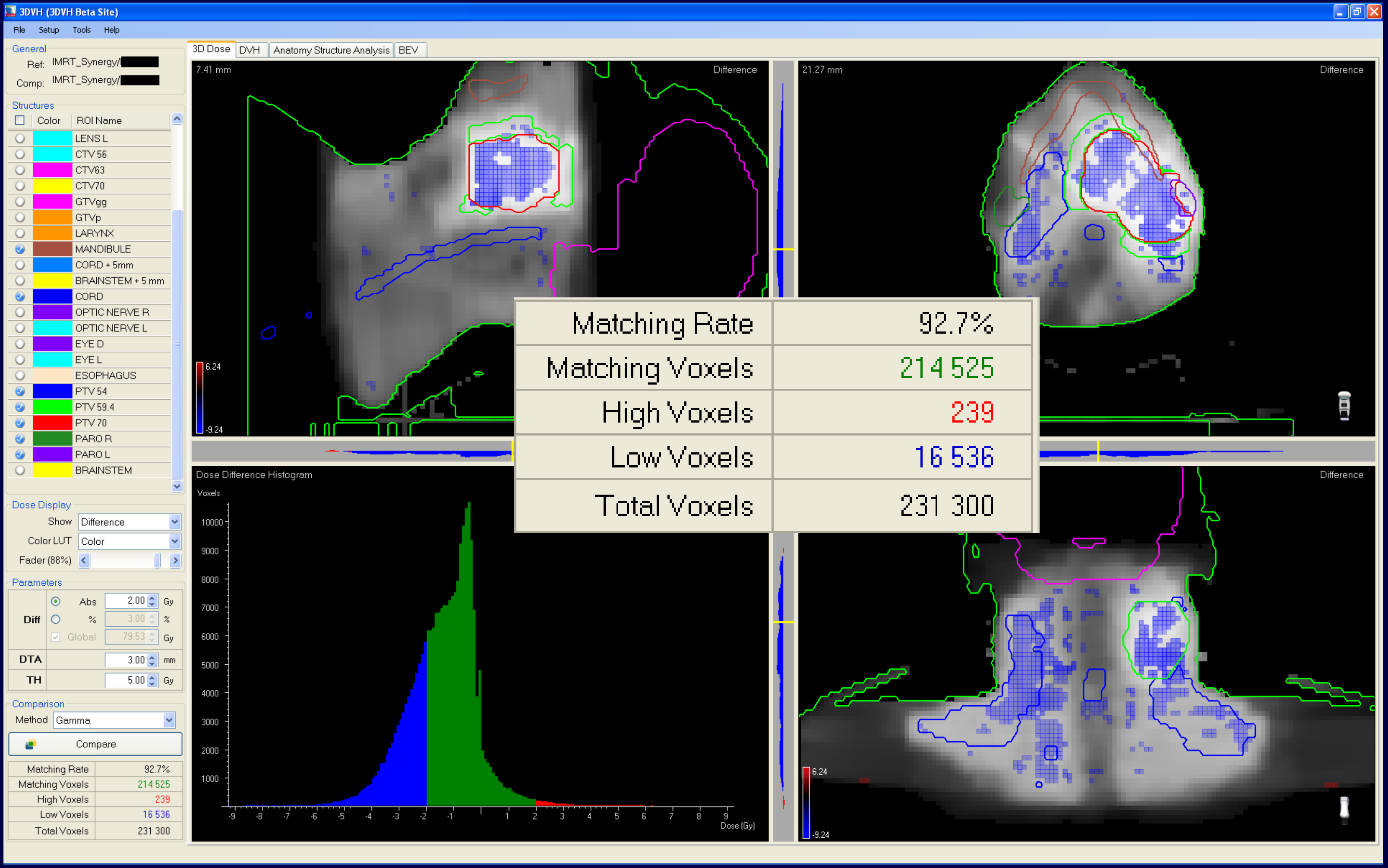
Clinical Issues

- Spatial resolution in evaluated distribution is important unless some type of interpolation is used
- Dose difference criterion is intuitive
- DTA criterion
  - Spatial uncertainty (measurements)
  - Spatial allowance (margins)
- How do we interpret γ failures?

γ failures

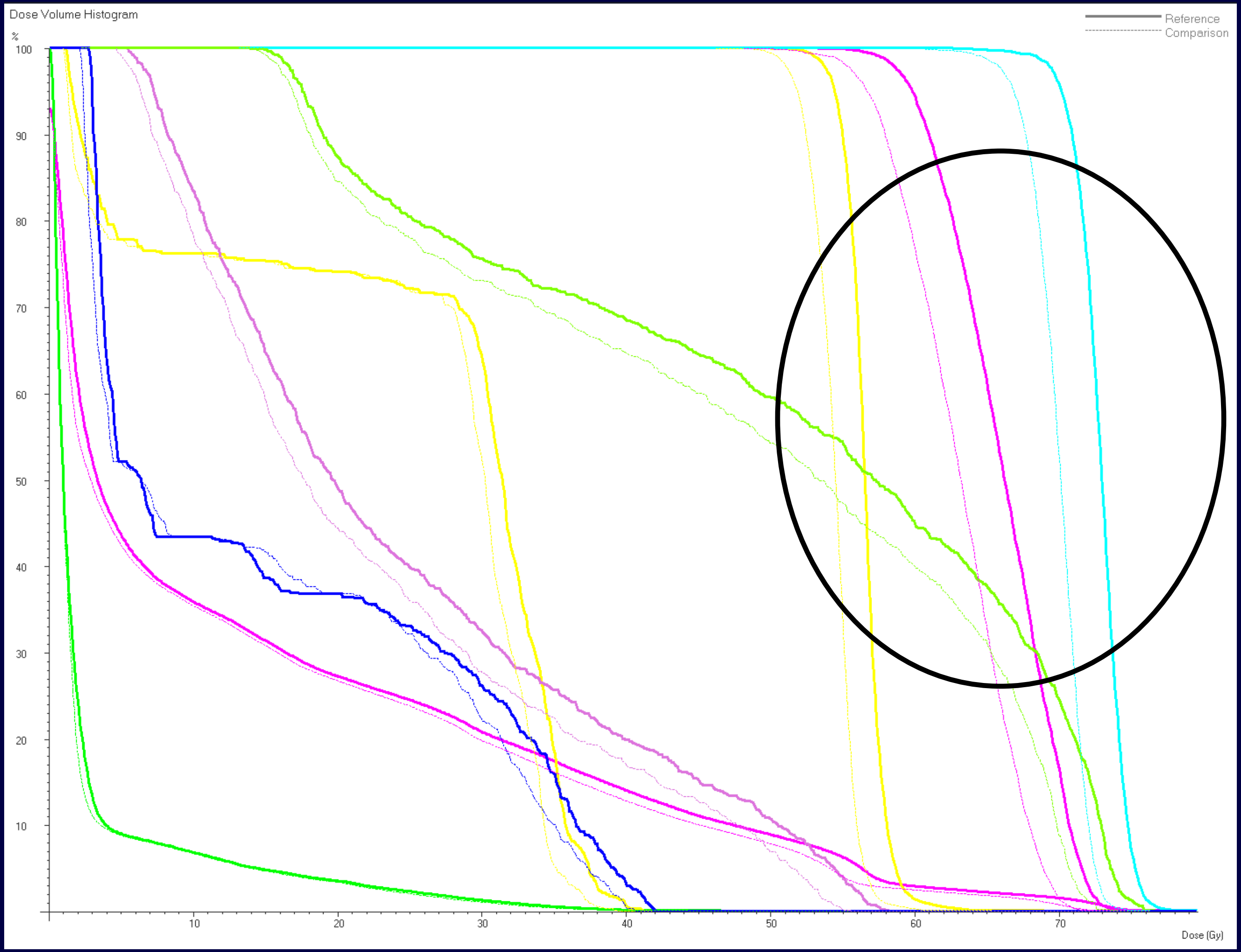
- 100% passing would be nice!
- Not practical
- Caution: γ tool should be used as an indicator of problems, not as a single indicator of plan quality





Thanks to Geneviève Jarry, HMR, Montreal

Target Volume Cold Spots



Thanks to Geneviève Jarry, HMR, Montreal

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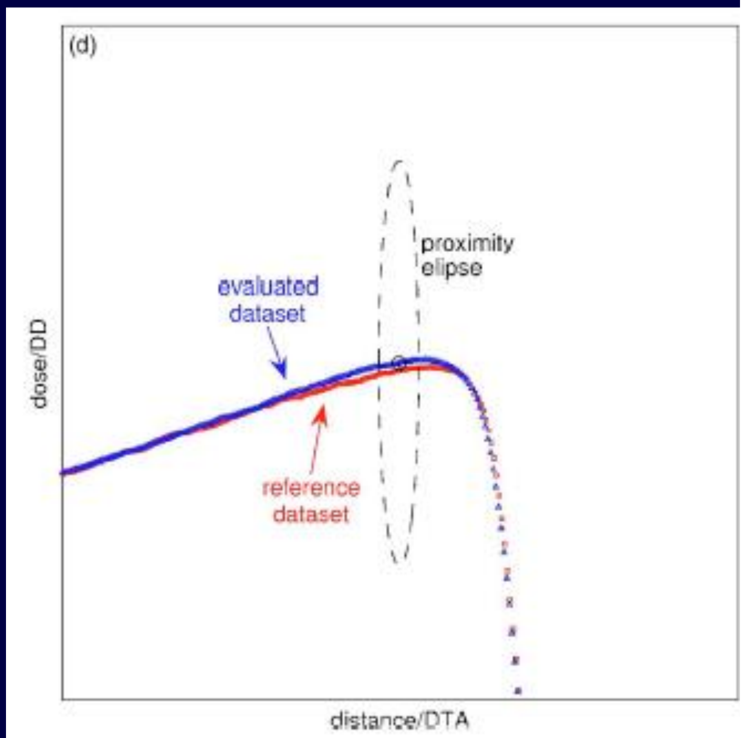
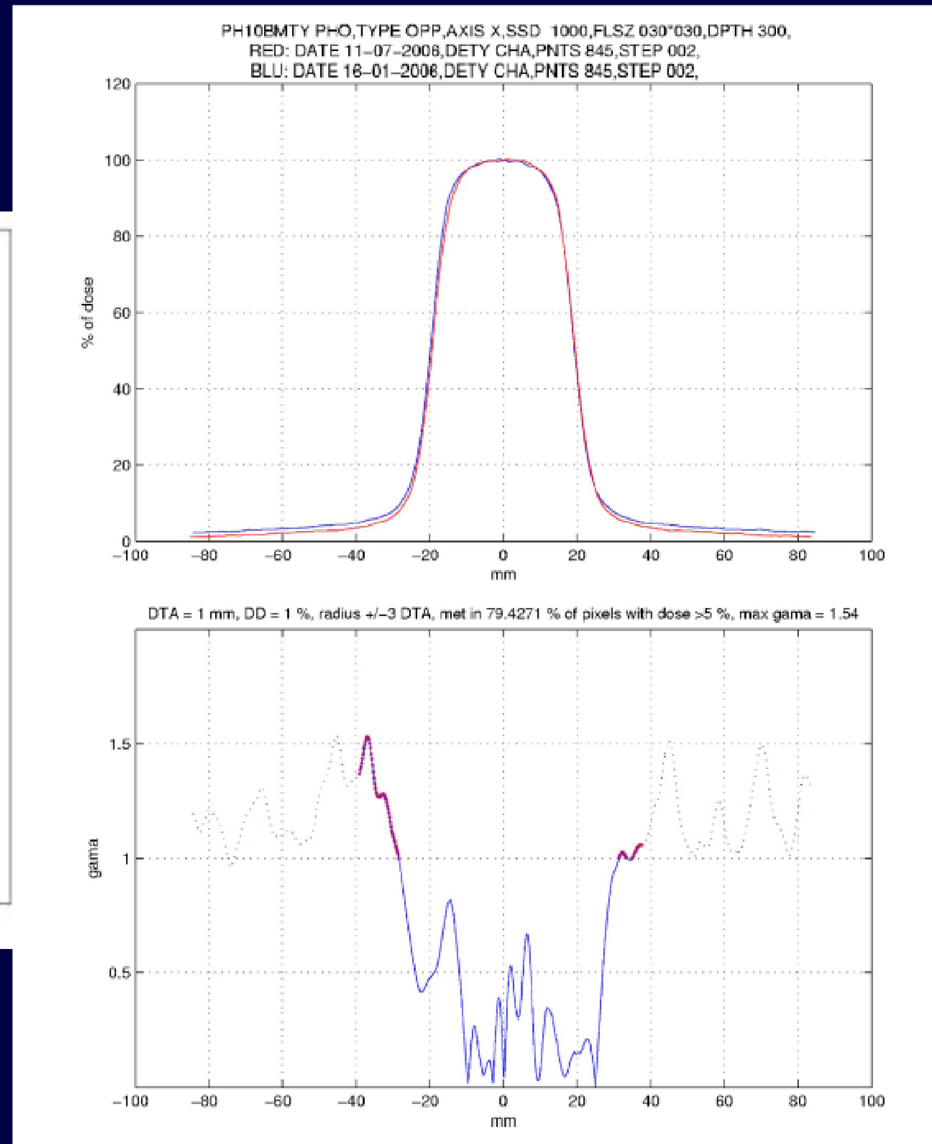
Normalization

- Gamma is sensitive to normalization
- Two otherwise identical dose distributions will have very poor γ distribution if they have different normalizations
- Renormalization may be necessary but be careful!

Other Applications

Hrbacek et al, Med Phys 34, 2917 (2007)





Criteria

- Spatially varying criteria (both dd and DTA
  - Anatomical (target versus muscle)
  - Dose (high versus low)
- This may be very useful with new back-projected and independently calculated 3D dose distributions
- Medically appropriate criteria will make interpretation of γ more straightforward

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γ Histograms

- γ histograms provide more information than just pass/fail percentages
- Maximum γ indicates magnitude of agreement
- Mean γ may also indicate relative quality of plan

Conclusions

- γ distribution is a powerful tool that aids in the evaluation of complex dose distributions
- γ is sensitive to noise; appropriate review should be made when noise is present
- Criteria should be more appropriately defined: spatially varying

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