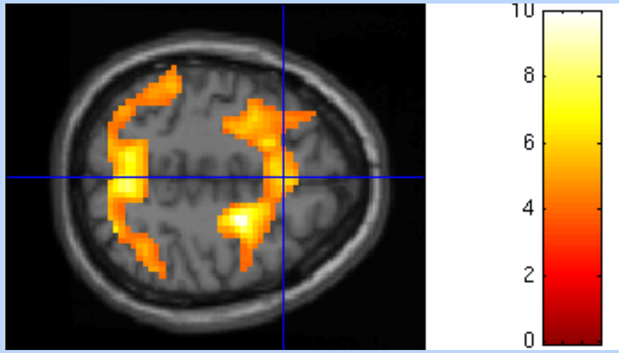
# Statistische Tests in SPM

## Teil I: Single subject Analyse (first Level)

### t-Kontraste und F-Kontraste




Michael Rose, SPM Kurs Hamburg 2018

# Übersicht

## 1. Level (Einzelpersonen)

Person 1: Modell (Regressoren)

Person 1: t- oder F-Kontraste

Person 2 …..Person n

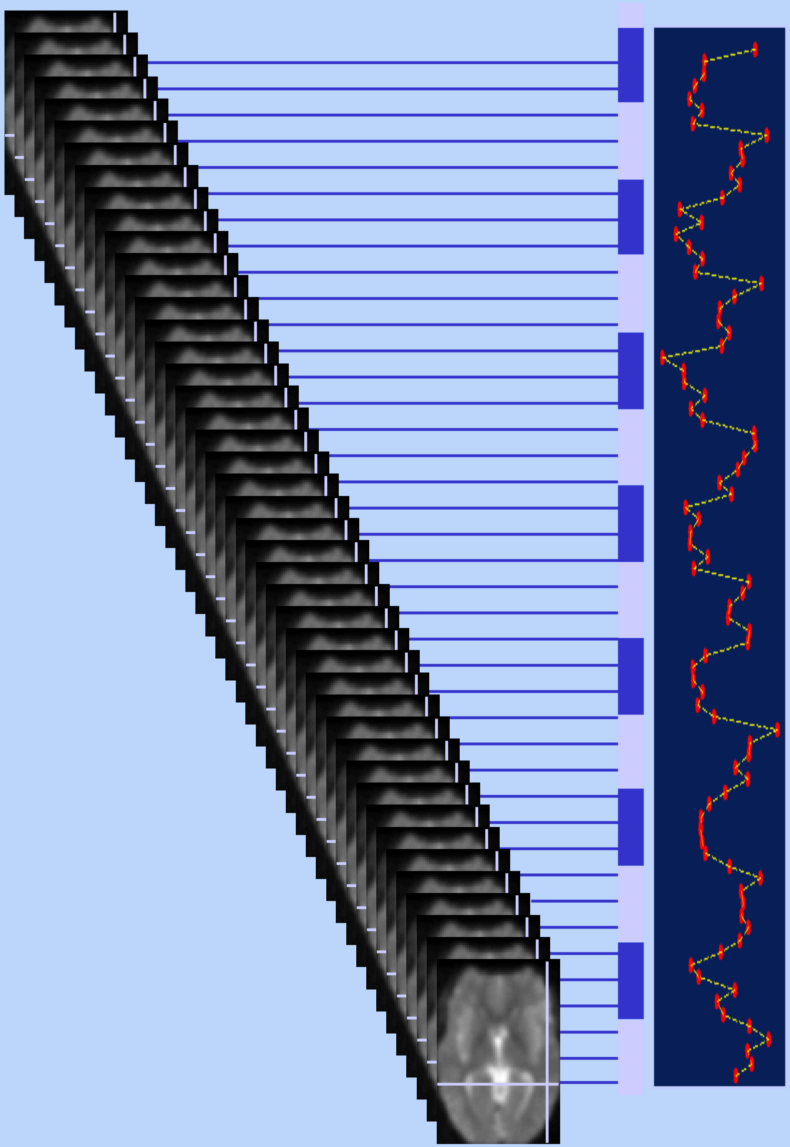
Ergebnisse

## 2. Level (Gruppe)

Modell für Gruppe (Regressoren)

Gruppe: t- oder F-Kontraste

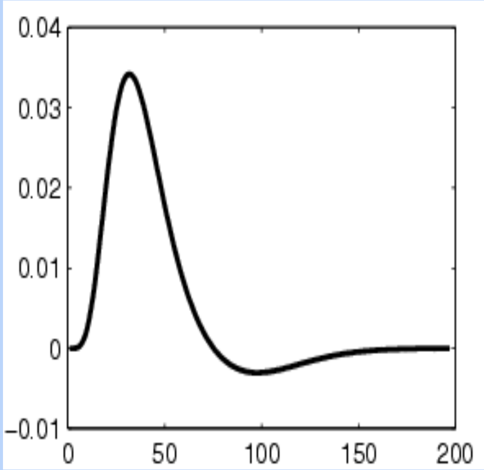
# 1. Level Daten für t-Statistik



Person 1:

Modell

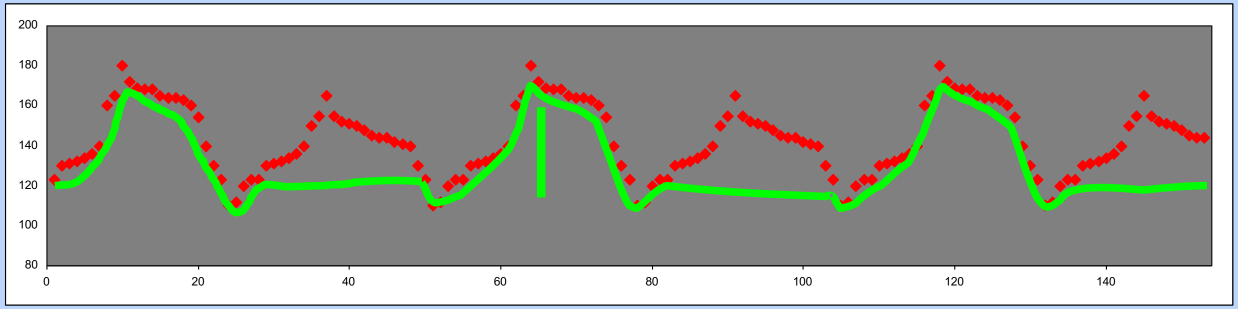
HRF



# 1. Level Daten für t-Statistik

Person 1

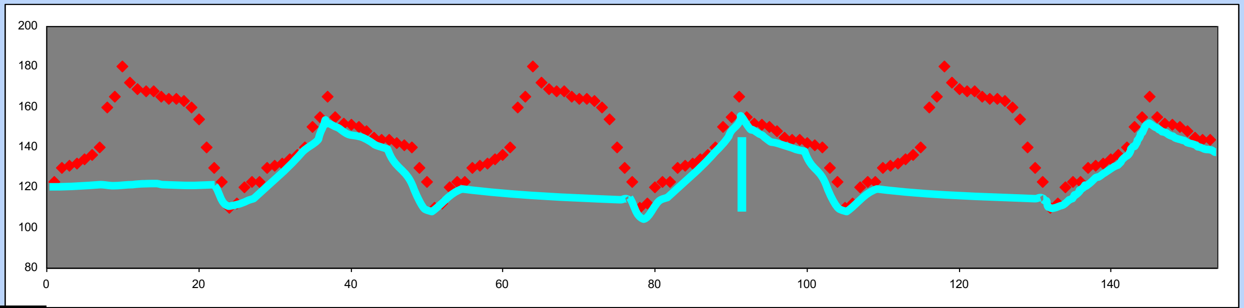
Bed. nb2



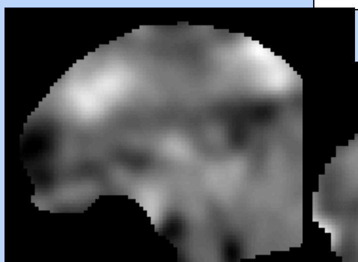

$\hat{\beta}_{nb2}$

Person 1

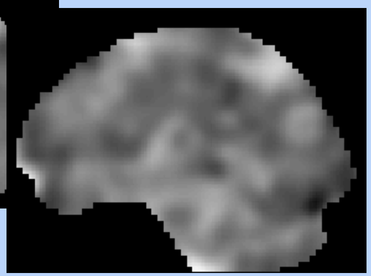
Bed. nb1



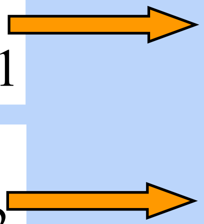

$\hat{\beta}_{nb1}$



beta_0001.img



beta_0002.img



$\hat{\beta}_{nb1}$ Geschätztes ß für nb1

$\hat{\beta}_{nb2}$ Geschätztes ß für nb2

# t-Kontraste

1. <u>Haupteffekte:</u>

Regressionskoeffizient ist größer als Null

**$\mathbf{H_1}$:** $\beta_{nb2} > 0$ **Test $\mathbf{H_0}$:** $\beta_{nb2} = 0$

2. <u>Differentielle Effekte:</u>

Bedingung nb2 führt zu einem größeren Effekt als nb1;

**$\mathbf{H_1}$:** $\beta_{nb2} > \beta_{nb1}$ **Test $\mathbf{H_0}$:** $\beta_{nb1} = \beta_{nb2}$

**t- Kontraste: gerichtete Hypothese**

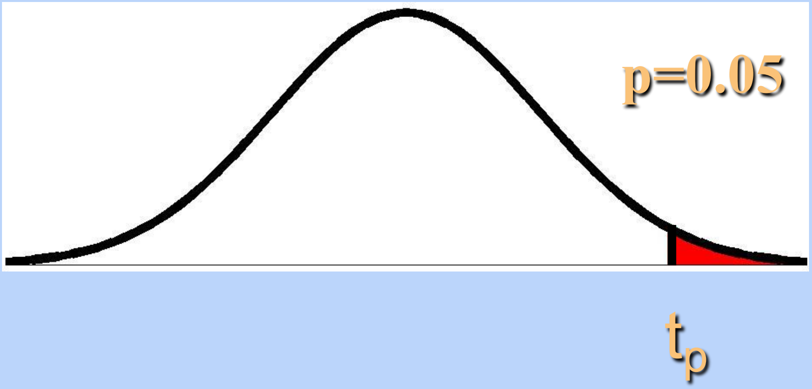
# t-Test

$$t = \frac{\hat{\beta}_{nb2} - \hat{\beta}_{nb1}}{\sqrt{\hat{\sigma}^2_{(\hat{\beta}_{nb2} - \hat{\beta}_{nb1})}}}$$

$$\beta_{nb1} = \beta_{nb2}$$

**Test $H_0$ : Mittelwerte sind eigentlich gleich, beobachtete Unterschiede sind zufällig**

t-Verteilung ($H_0$)

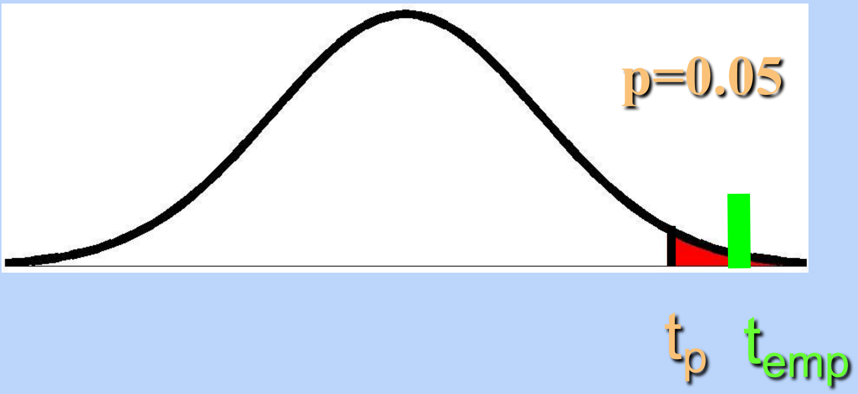



**p= Irrtumswahrscheinlichkeit**
**t= t-Wert zu p**

**Form der t-Verteilung von Freiheitsgraden (df) abhängig**

# t-Test

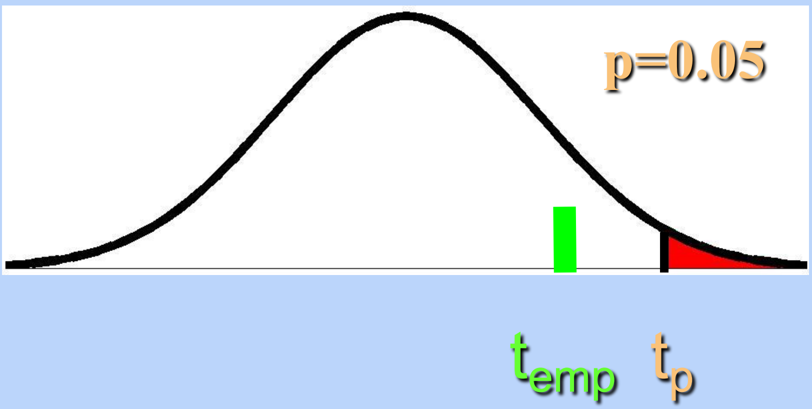
t-Verteilung ($H_0$)




$\cancel{\beta_{nb1} = \beta_{nb2}}$ $\beta_{nb2} > \beta_{nb1}$

**$H_0$ wird verworfen!**

**(Mittelwerte sind sign. verschieden)**




$\beta_{nb1} = \beta_{nb2}$ $\cancel{\beta_{nb2} > \beta_{nb1}}$

**$H_0$ kann nicht verworfen werden!**

**(Mittelwerte sind nicht sign. verschieden)**

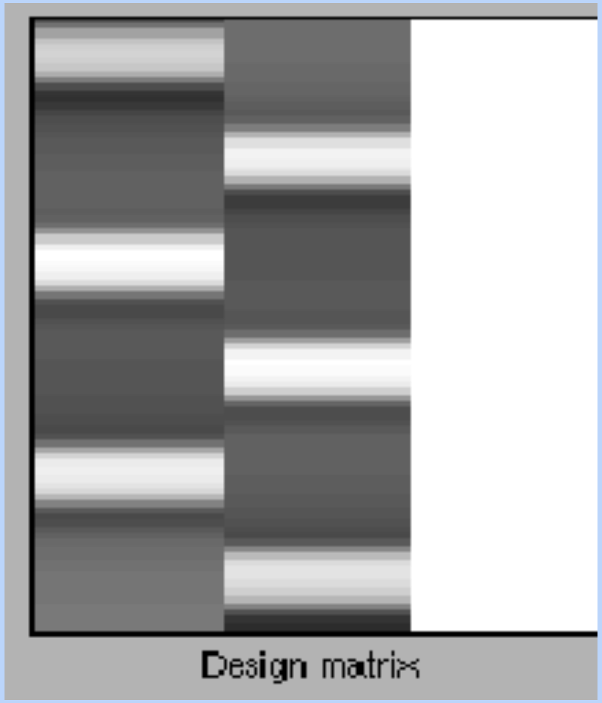
# t-Kontraste

$$t = \frac{\hat{\beta}_{nb2} - \hat{\beta}_{nb1}}{\sqrt{\hat{\sigma}^2_{(\hat{\beta}_{nb2} - \hat{\beta}_{nb1})}}}$$

$$D = 1 * \hat{\beta}_{nb2} + (-1) * \hat{\beta}_{nb1}$$

$c_1$ $c_2$

Kontrastgewichtungen




Kontraste: Einzelvergleiche bestimmter Bedingungen

entstehen aus Hypothesen

$$c' = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} * \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_0 \end{bmatrix}$$

# Kontraste für mehrere Parameter

Beispiel: Ist Mittelwert der 3 Experimentalbedingungen größer als der Effekt der Kontrollbedingung ?

$$D = (\hat{\beta}_1 + \hat{\beta}_2 + \hat{\beta}_3)/3 - \hat{\beta}_4$$

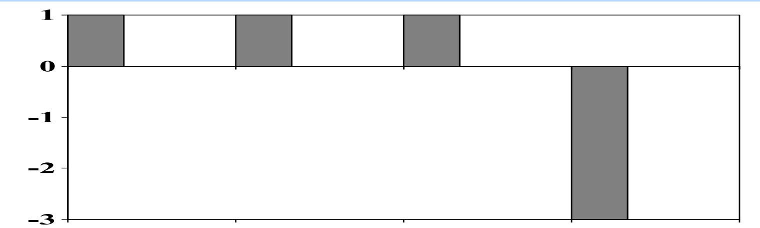
$$D = 0.33 * \hat{\beta}_1 + 0.33 * \hat{\beta}_2 + 0.33 * \hat{\beta}_3 + (-1) * \hat{\beta}_4$$

$C_1=1/3$ $C_2=1/3$ $C_3=1/3$ $C_4=(-1)$

Führt zur gleichen Statistik wie:

$C_1=1$ $C_2=1$ $C_3=1$ $C_4=(-3)$

$$D = 1 * \hat{\beta}_1 + 1 * \hat{\beta}_2 + 1 * \hat{\beta}_3 + (-3) * \hat{\beta}_4$$



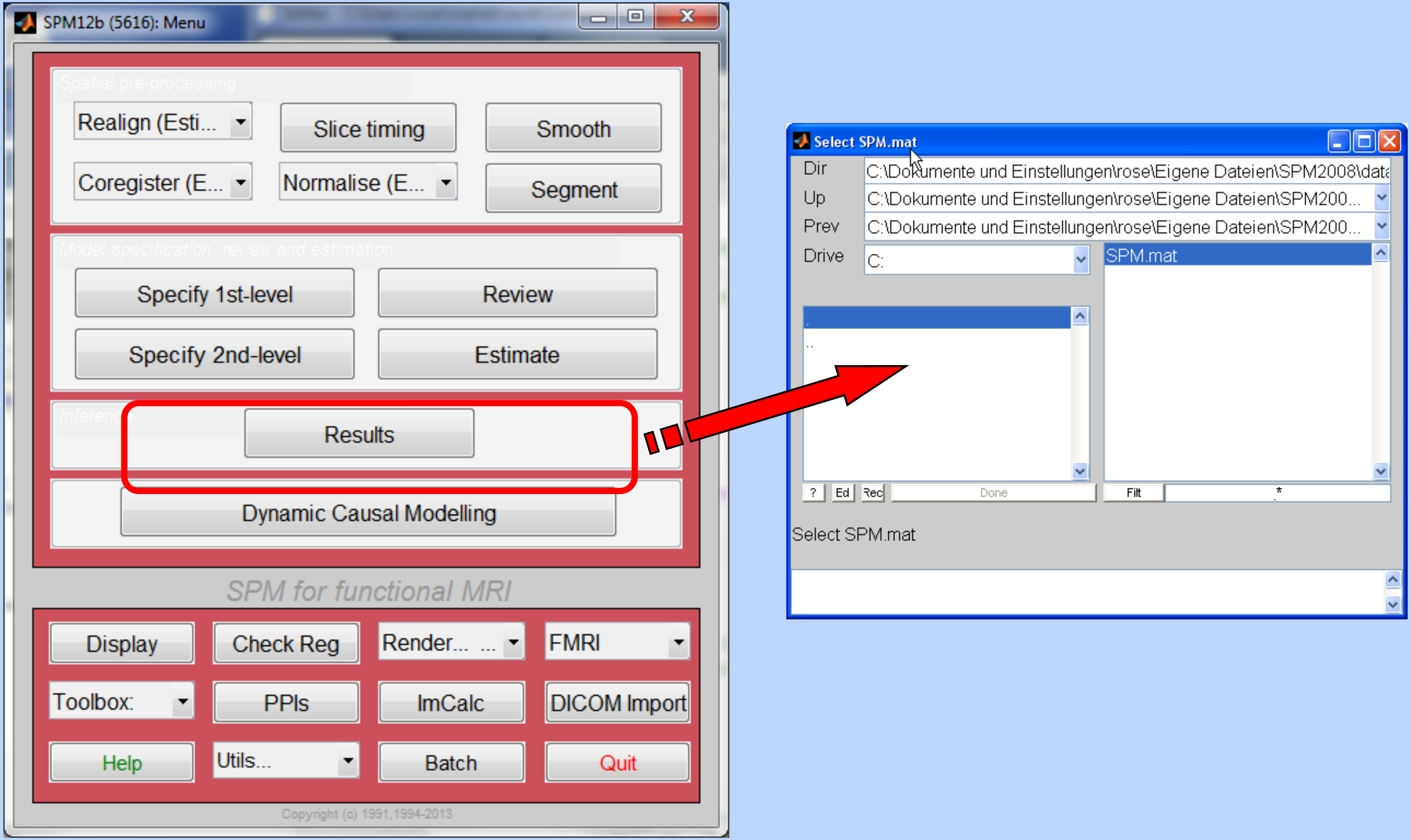

$$D = \sum_i c_i * \hat{\beta}_i$$

wobei

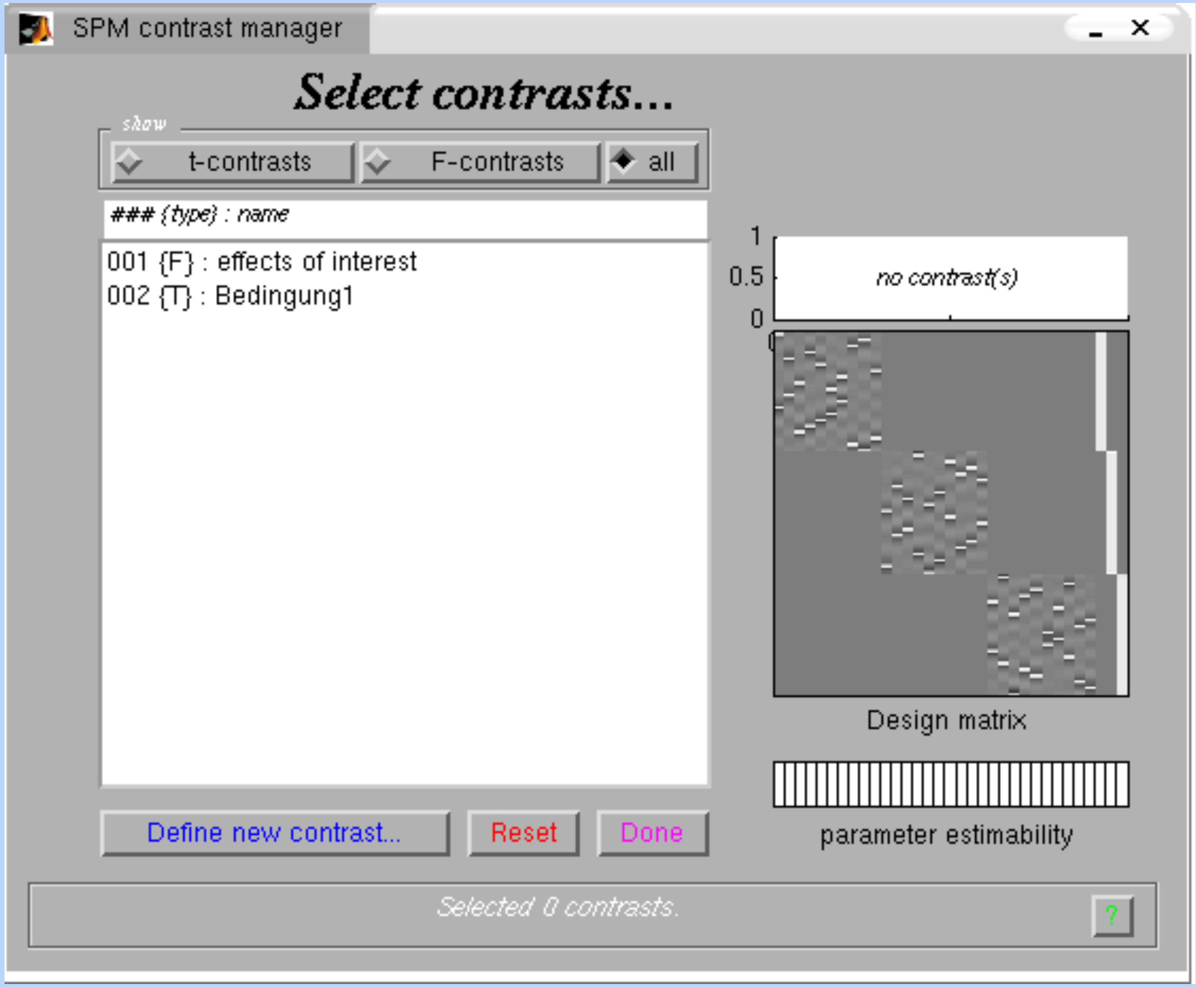
$$\sum_i c_i = 0$$

Für Differenzen

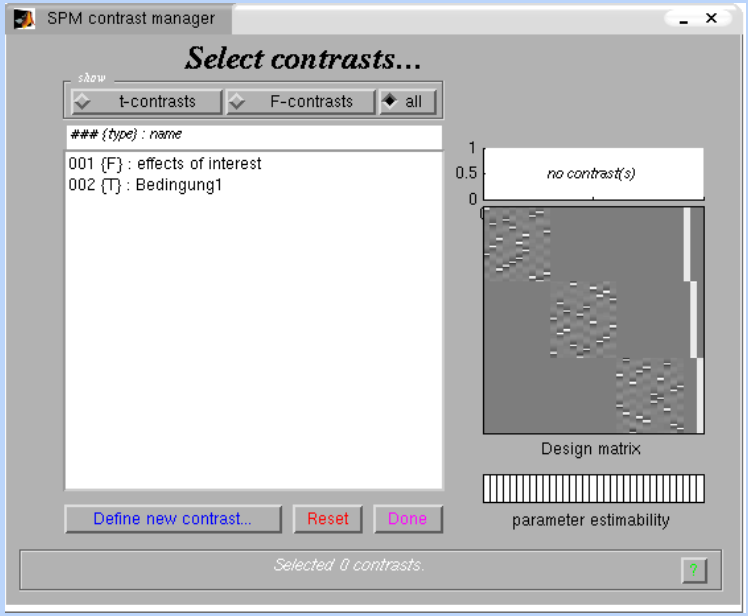
# Kontrast-Manager

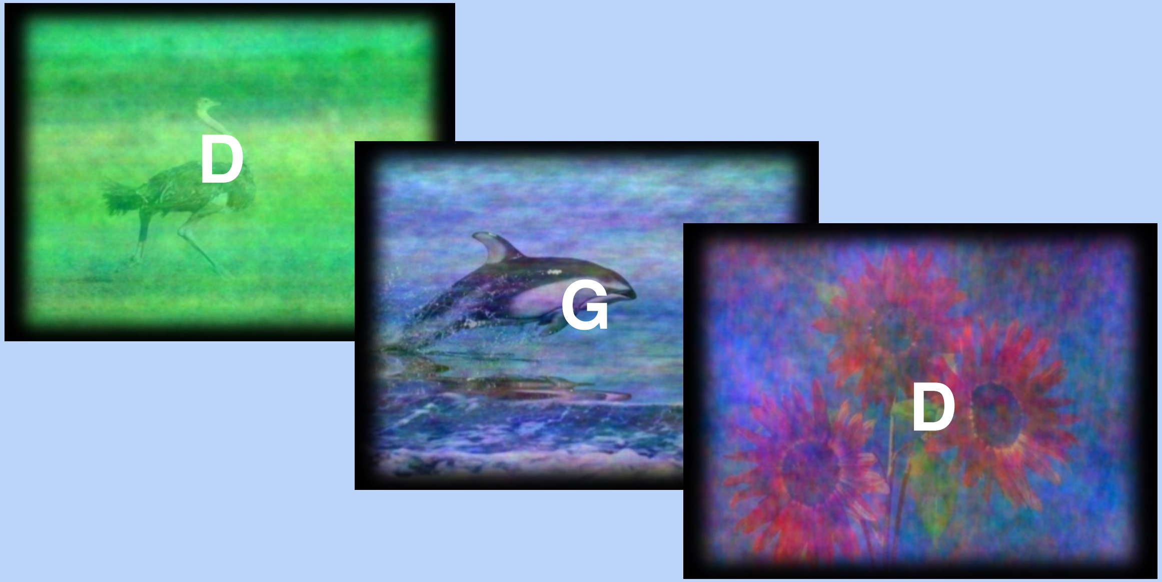


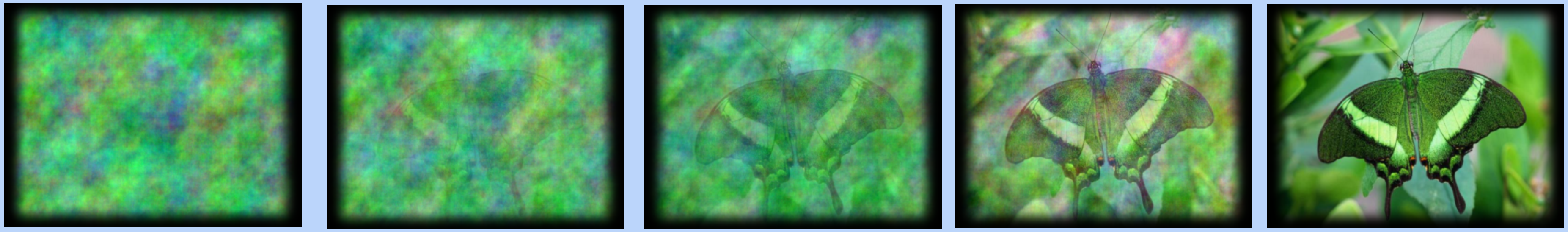
# Kontrast-Manager



# Kontrast-Manager

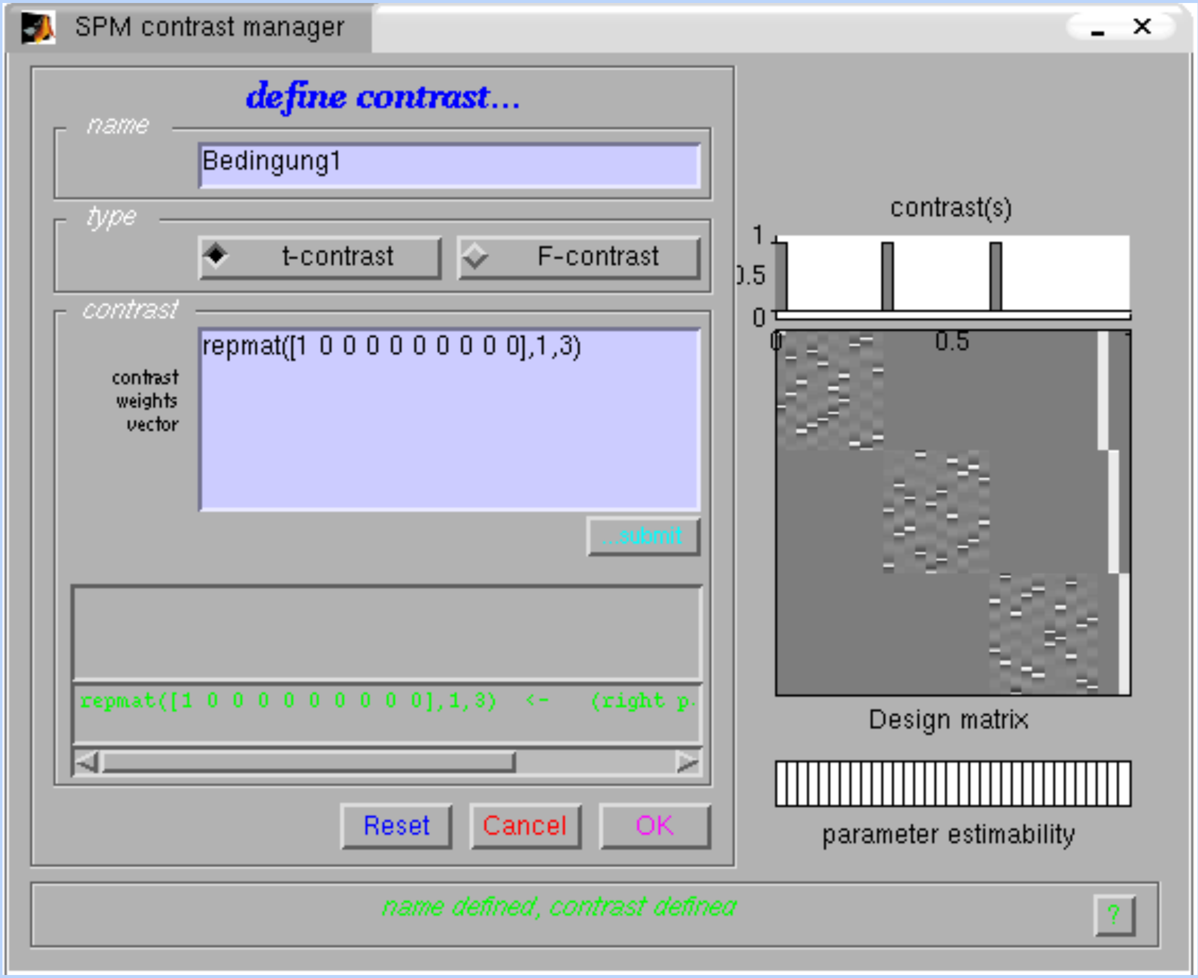


**N-back 1 (1:5) oder 2 (6:10)**

# Haupteffekte



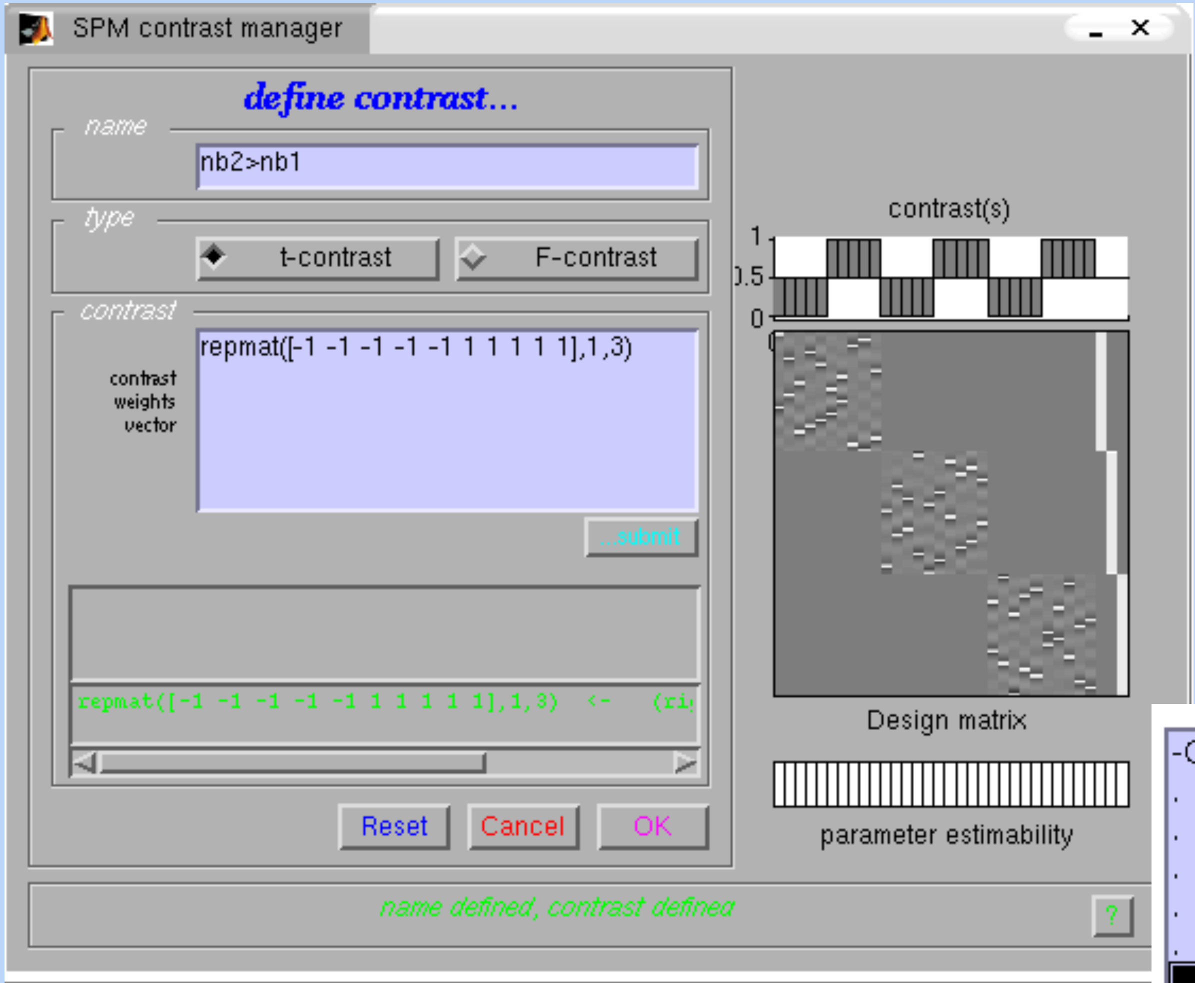

$$\overline{\beta}_{b1} > 0$$

$$t = \frac{\overline{\beta}_{b1}}{\sqrt{\hat{\sigma}^2_{\hat{\beta}_1}}}$$

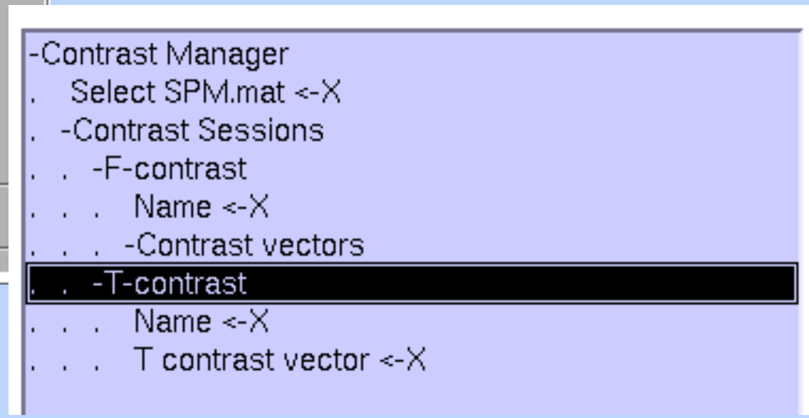
Wichtig: Summe wird gebildet durch Linearkombination

**C= 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0**

# Differentielle Kontraste




$$D = 1 * \bar{\beta}_{nb2} + (-1) * \bar{\beta}_{nb1}$$

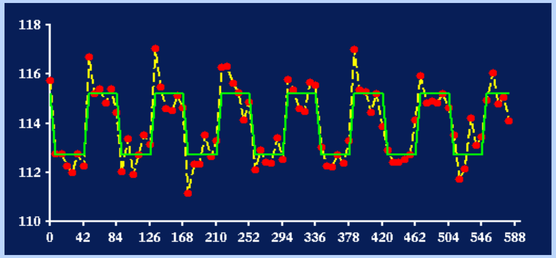



**C= -1 -1 -1 -1 -1 1 1 1 1 1 1 .........**

# Fehler-Varianz

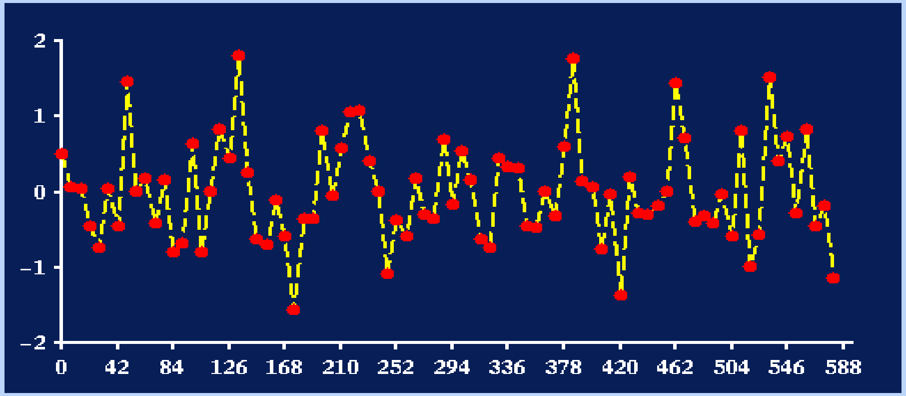
$$t = \frac{\hat{\beta}_{nb2} - \hat{\beta}_{nb1}}{\sqrt{\hat{\sigma}^2_{(\hat{\beta}_{nb2} - \hat{\beta}_{nb1})}}}$$

$$\hat{\sigma}^2_{(\hat{\beta}_{nb2} - \hat{\beta}_{nb1})} = f\left(\hat{\sigma}^2, c, X\right)$$




**fitted box-car**

**Residuen**



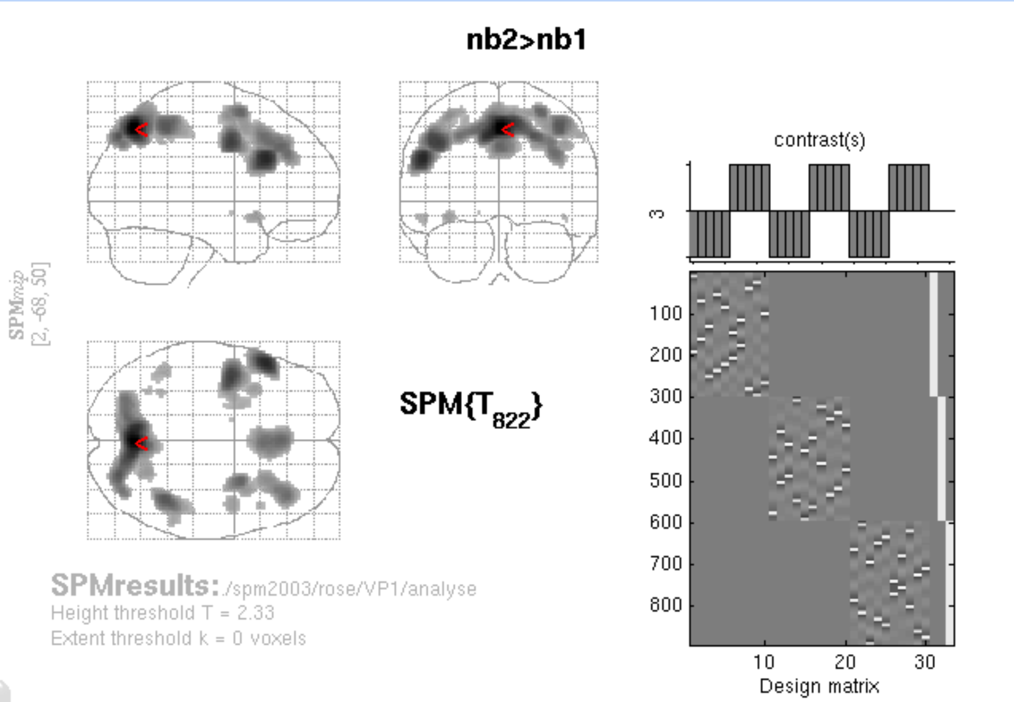

Residuen nach Anpassung des <u>gesamten</u> Modells (ResMS.img)

$$\frac{\sum \text{der quadrierten Residuen}}{\text{Anzahl der Datenpunkte minus der Anzahl der Regressoren}} = \hat{\sigma}^2$$

# SPM{t} - Map

Nback 2 > Nback 1

Jeder Kontrast ergibt einen t-Wert in jedem einzelnen Voxel, z.B. 5.20




*df = n-s*
*n: Meßpunkte*
*s:Parameter*

Statistics: *p-values adjusted for search volume*

| set-level | | cluster-level | | | voxel-level | | | | | x,y,z {mm} | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $p$ | $c$ | $p_{corrected}$ | $k_E$ | $p_{uncorrected}$ | $p_{FWE-corr}$ | $p_{FDR-corr}$ | $T$ | $(Z_{\equiv})$ | $p_{uncorrected}$ | | | |
| **0.998** | **14** | **0.004** | **2216** | **0.000** | **0.003** | **0.006** | **5.20** | **5.16** | **0.000** | **2** | **-68** | **50** |
| | | | | | 0.517 | 0.036 | 3.83 | 3.82 | 0.000 | 24 | -80 | 44 |
| | | | | | 0.777 | 0.055 | 3.60 | 3.59 | 0.000 | -14 | -72 | 50 |
| | | **0.557** | **545** | **0.030** | **0.062** | **0.011** | **4.51** | **4.48** | **0.000** | **-52** | **22** | **28** |
| | | **0.241** | **817** | **0.010** | **0.236** | **0.021** | **4.13** | **4.10** | **0.000** | **-42** | **-2** | **40** |
| | | | | | 0.999 | 0.141 | 3.03 | 3.02 | 0.001 | -34 | -4 | 58 |
| | | | | | 1.000 | 0.261 | 2.62 | 2.61 | 0.005 | -24 | 4 | 54 |
| | | **0.563** | **541** | **0.031** | **0.579** | **0.040** | **3.78** | **3.76** | **0.000** | **42** | **-48** | **52** |
| | | | | | 1.000 | 0.237 | 2.68 | 2.68 | 0.004 | 56 | -42 | 46 |
| | | **0.271** | **781** | **0.012** | **0.743** | **0.052** | **3.64** | **3.62** | **0.000** | **28** | **4** | **62** |
| | | | | | 0.870 | 0.066 | 3.50 | 3.49 | 0.000 | 38 | 36 | 32 |
| | | | | | 0.969 | 0.089 | 3.31 | 3.30 | 0.000 | 34 | 32 | 40 |
| | | **0.221** | **843** | **0.009** | **0.767** | **0.054** | **3.61** | **3.60** | **0.000** | **0** | **32** | **38** |
| | | | | | 0.881 | 0.068 | 3.49 | 3.47 | 0.000 | 2 | 22 | 42 |

# F - Kontraste

Wieviel Varianz erklären die ausgewählten Bedingungen?

Auswahl über Kontraste

1

0 1

0 0 1

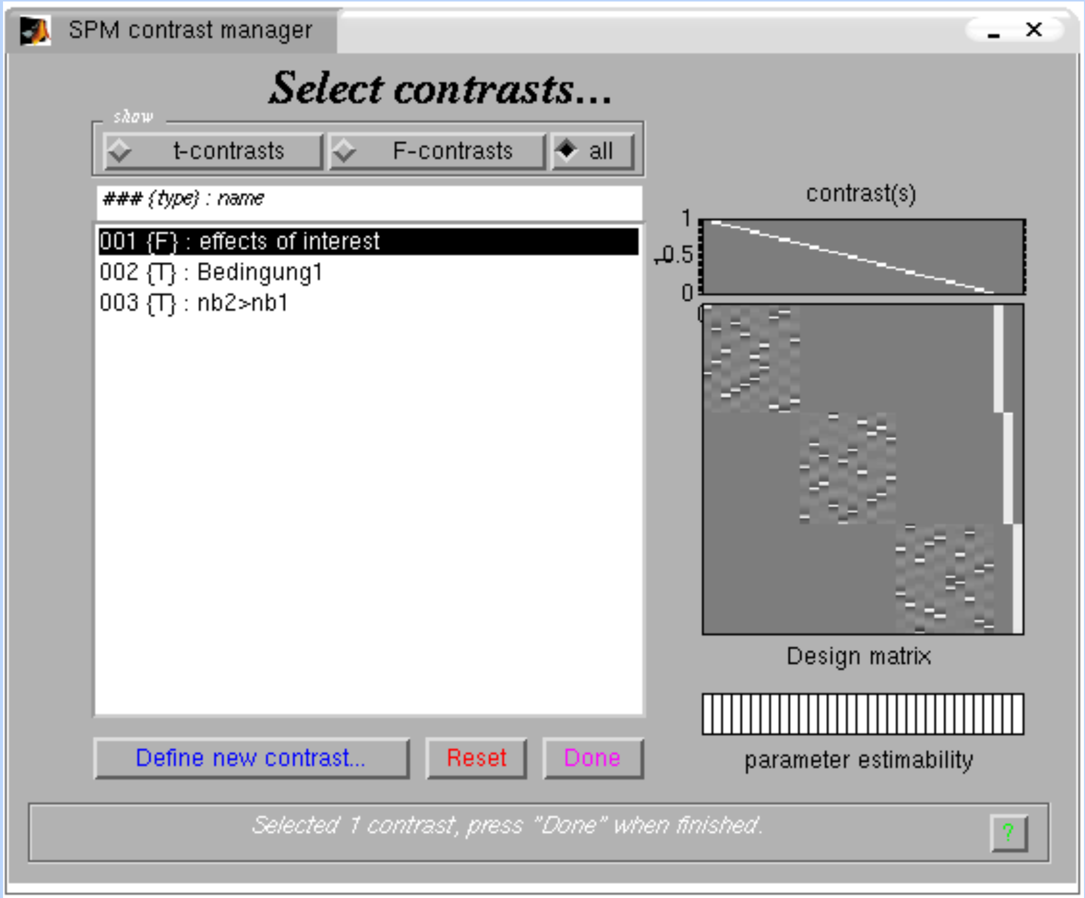
0 0 0 1

0 0 0 0 1

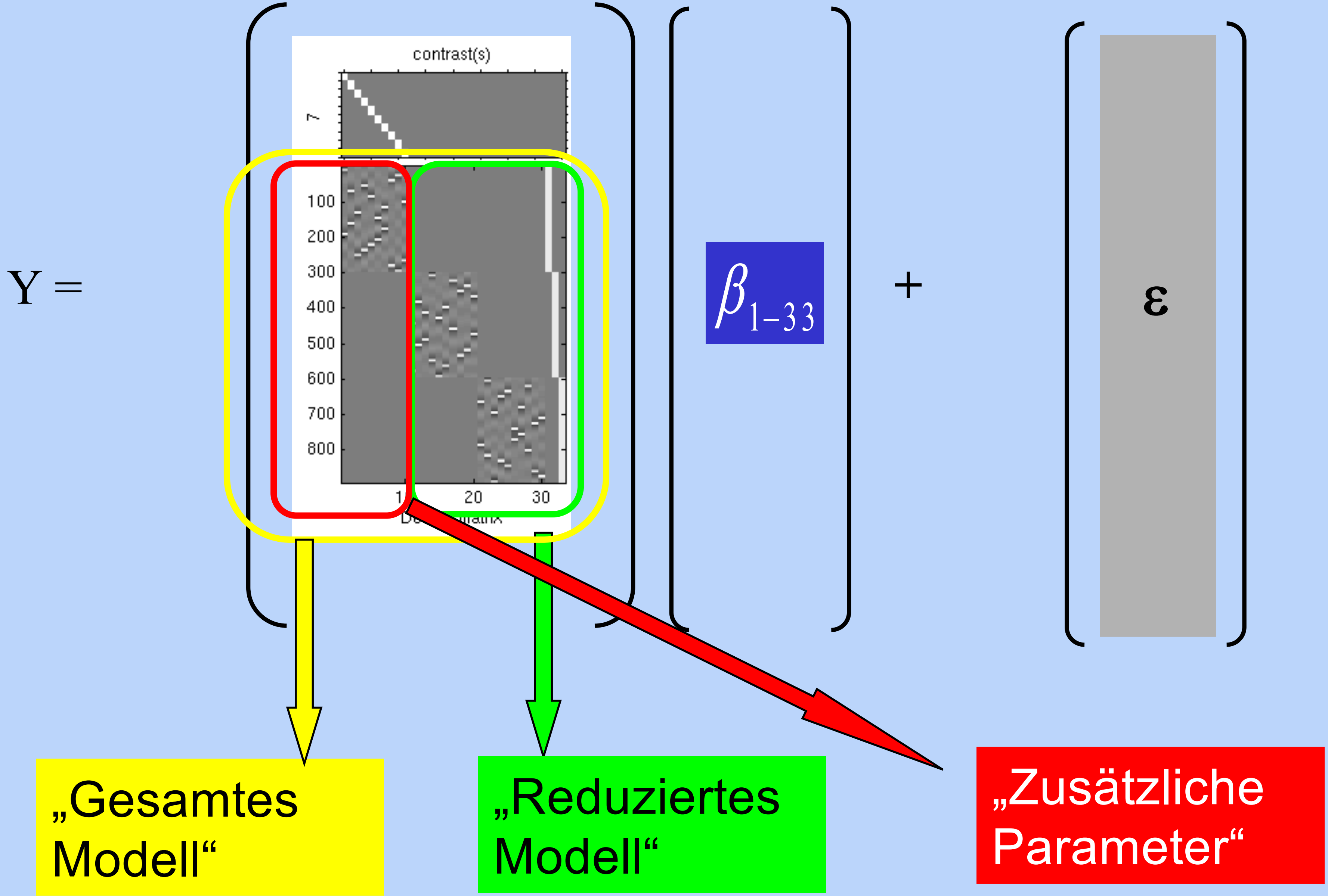
0 0 0 0 0 1

0 0 0 0 0 0 1

0 0 0 0 0 0 0 1........

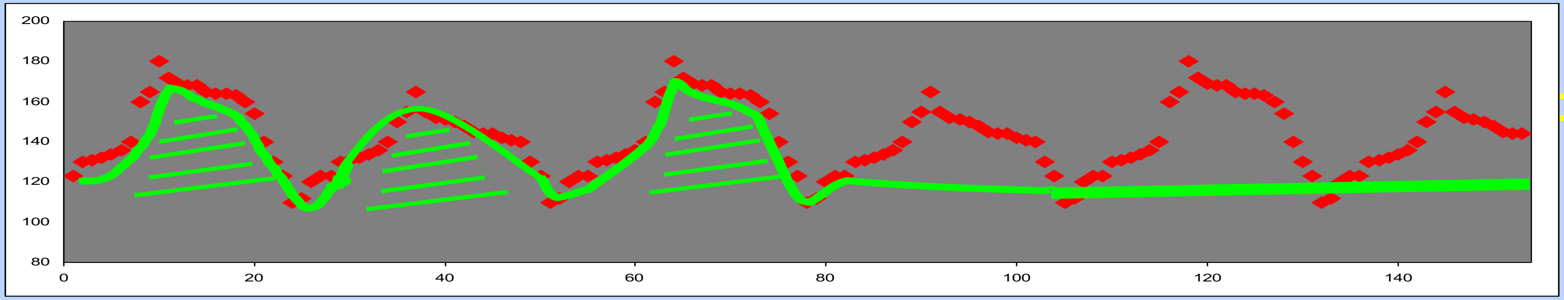


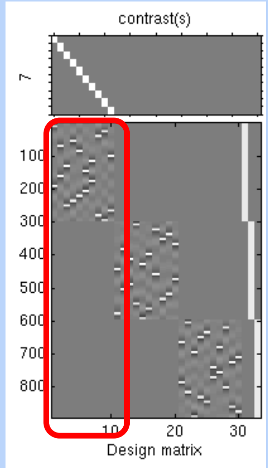
# F - Kontraste



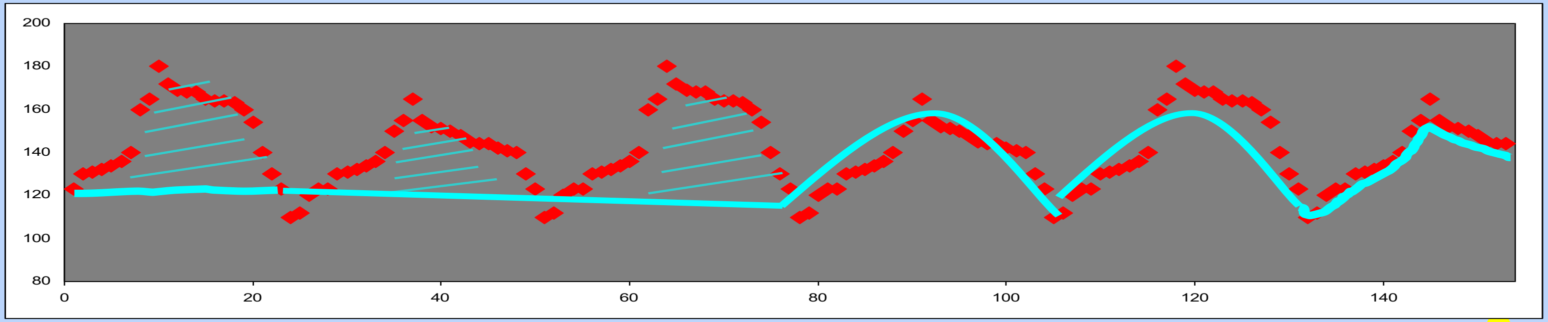
Y =
contrast(s)
100
200
300
400
500
600
700
800
20
30
β1–33
+
ε
„Gesamtes Modell“
„Reduziertes Modell“
„Zusätzliche Parameter“

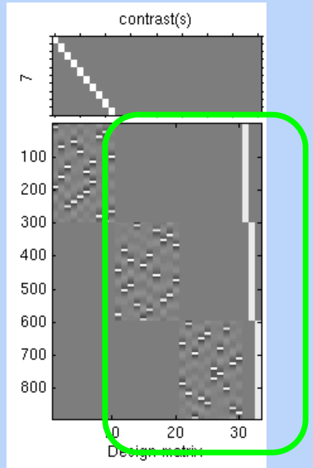
## „Zusätzliche Varianz“



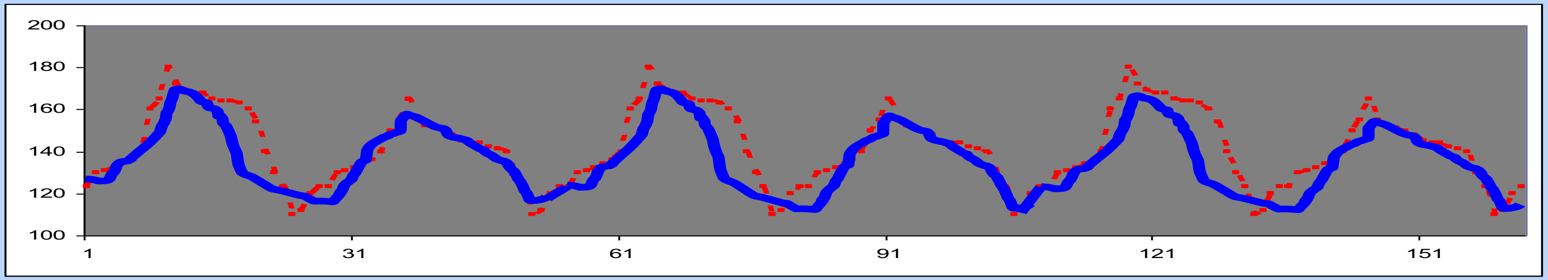



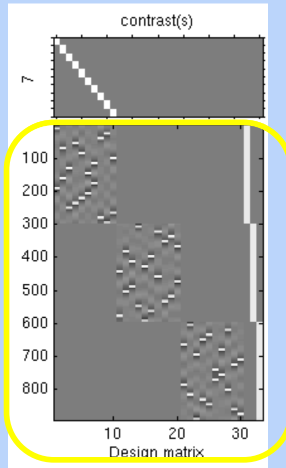

## „Fehlervarianz des reduzierten Modells“




## „Fehlervarianz des gesamten Modells“



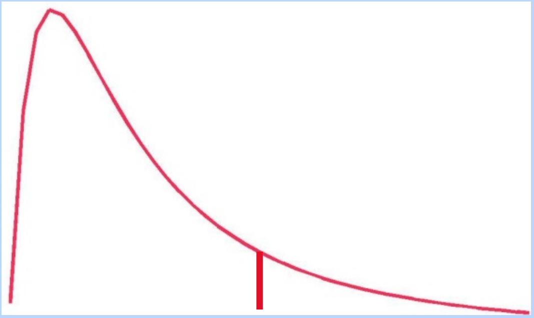
# F - Kontraste

Zusätzliche Varianz = Fehlervarianz des reduzierten Modells - Fehlervarianz des Gesamtmodells

(ess_*.img = extra-sum-of-squares des entsprechenden F-Kontrastes)

$$F = \frac{(RSS^2_{\mathrm{Re}d} - \hat{\sigma}^2)/(s - s_{\mathrm{Re}d})}{\hat{\sigma}^2/(n-s)}$$

*df(Zähler)= s - $s_{Red}$*
*df(Nenner)= n-s*

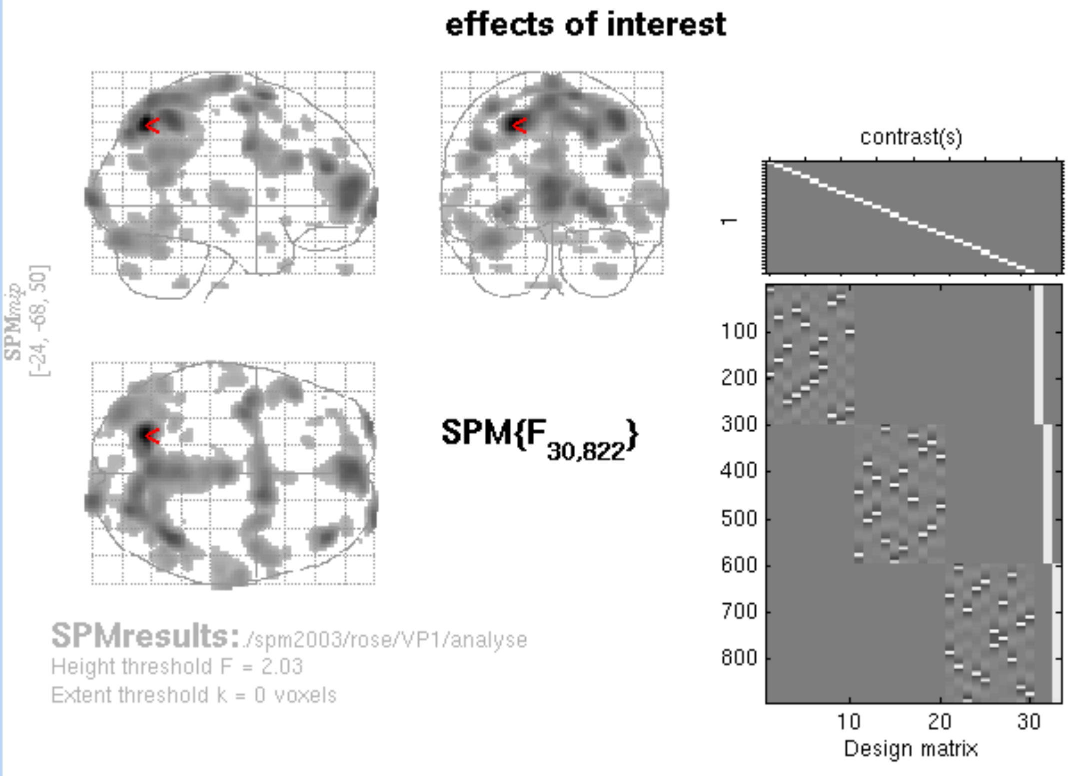


$F_p$

# SPM{F}-Map

Jeder Kontrast ergibt einen F-Wert in jedem einzelnen Voxel, z.B. 8.3

$$F = \frac{(RSS^2_{\mathrm{Red}} - \hat{\sigma}^2)/(s - s_{\mathrm{Red}})}{\hat{\sigma}^2/(n-s)}$$




**Statistics:** *p-values adjusted for search volume*

| set-level | | cluster-level | | | voxel-level | | | | | x,y,z {mm} | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| p | c | p corrected | $k_E$ | p uncorrected | p FWE-corr | p FDR-corr | F | ($Z_\equiv$) | p uncorrected | | | |
| **0.000** | **49** | | **4292** | | **0.000** | **0.000** | **8.30** | **Inf** | **0.000** | **-24** | **-68** | **50** |
| | | | | | 0.000 | 0.000 | 5.66 | Inf | 0.000 | 40 | -52 | 52 |
| | | | | | 0.000 | 0.000 | 5.15 | Inf | 0.000 | 22 | -78 | 44 |
| | | | **1672** | | **0.000** | **0.000** | **5.33** | **Inf** | **0.000** | **-2** | **58** | **12** |
| | | | | | 0.000 | 0.000 | 5.15 | Inf | 0.000 | 0 | 56 | 2 |
| | | | | | 0.015 | 0.000 | 2.94 | 4.96 | 0.000 | -20 | 58 | 22 |
| | | | **1721** | | **0.000** | **0.000** | **5.14** | **Inf** | **0.000** | **14** | **4** | **68** |
| | | | | | 0.000 | 0.000 | 4.56 | 7.49 | 0.000 | -36 | -6 | 60 |
| | | | | | 0.000 | 0.000 | 4.54 | 7.46 | 0.000 | -44 | -4 | 36 |
| | | | **299** | | **0.000** | **0.000** | **5.07** | **Inf** | **0.000** | **40** | **44** | **22** |
| | | | **631** | | **0.000** | **0.000** | **4.97** | **Inf** | **0.000** | **18** | **-100** | **2** |
| | | | | | 0.952 | 0.004 | 2.25 | 3.58 | 0.000 | -6 | -88 | -16 |
| | | | **156** | | **0.000** | **0.000** | **4.85** | **Inf** | **0.000** | **28** | **72** | **-4** |

# F – Kontraste

C= [0 1 0 …;
0 0 1…];

Parameter der 2. ODER 3. Spalte signifikant verschieden von Null

C= [0 1 1 ];

Summe der Parameter der 2. UND 3. Spalte signifikant verschieden von Null

Differentielle F-Tests sind zweiseitig also nicht gerichtet:

1 -1 entspricht -1 1

# Zusammenfassung

In jedem Voxel:

- Gerichteter Unterschied zwischen den Schätzungen der Parameter (t-Kontrast)

$\Rightarrow$ SPM{t}-Map

- Nicht-gerichteter Unterschied zwischen Varianzschätzern (F-Kontrast)

$\Rightarrow$ SPM{F}-Map